THE MEASUREMENT OF THE ANODE CIRCUIT IMPEDANCES AND MUTUAL CONDUCTANCES OF THERMIONIC VALVES

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ABSTRACT. The paper describes the application of the Wheatstone Bridge to the measurement of the anode circuit admittance or impedance, and the mutual conductance of a valve under actual operating conditions. Current of telephonic frequency is used. The measurements can be made for grid bias of any desired value, and both methods can be made direct-reading. The results of measurements made on a few typical valves are given, and it is shown that although both anode circuit resistance and mutual conductance vary very considerably with the grid bias, the product of the two, which gives the voltage factor of the valve, is approximately constant. The anode circuit admittance consists of a conductance associated with a comparatively small capacity, but this capacity is larger than the inter-electrode capacities of the valve when the filament is cold. The increase in the effective values of the inter-electrode capacities is explained by the presence of the space charge, which also has the effects of making these capacities vary with the frequency and of giving them a comparatively high power factor, especially at low frequencies.

§ 1. INTRODUCTION

VARIOUS methods of measuring the characteristic constants of thermionic valves have already been described*. Approximate values may be obtained from the static characteristic curves, but in order to obtain values under actual working conditions, alternating current must be used. For precision measurements at telephonic frequencies, circuits of the Wheatstone bridge type have much to recommend them. With proper technique, errors due to earth capacities can be successfully avoided even at high frequencies, and in this respect they are preferred by the author to the various alternating current methods of the potentiometer type which have already been described for the measurement of the constants of

* See for example Van der Bijl, *The Thermionic Vacuum Tube*, pp. 197–205; Eccles, *Continuous Wave Wireless Telegraphy*, pp. 386–393.

valves. The present paper describes the application of this network to the measurement of the mutual conductances and anode circuit impedances of thermionic valves.

In considering the behaviour of the valve with respect to alternating current, we may regard it as a device with three terminals—the anode A, the grid G, and the filament F, Fig. 1. A current i_g (the "grid current") is passed through the valve by way of the terminals G and F, and a current I_a (the "anode current") is taken from it by way of the terminals A and F. Let v be the p.d. between G and F, and V that between A and F. Then the important characteristic constants are:

(a) The grid circuit admittance, or input admittance $\partial i_g/\partial v$.
(b) The anode circuit admittance, or output admittance $\partial I_a/\partial V$.
(c) The mutual admittance $\partial I_a/\partial v$.

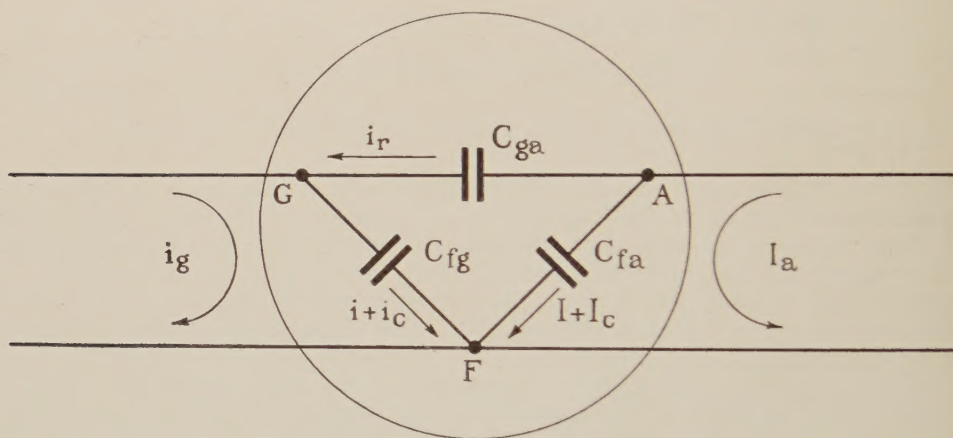The reciprocals of these give the corresponding impedances.

Fig. 1. Schematic diagram of valve.

These quantities are nearly but not quite pure conductances. The actual current between the valve electrodes is partly thermionic (i and I) and partly capacity current (i_c, I_c, and i_r, Fig. 1), and by a simple application of Kirchhoff's Laws to Fig. 1 it may be shown that in the usual vector notation

$$\partial i_g/\partial v = \partial i/\partial v + jw\,(C_{fg} + C_{ga}) = \partial i/\partial v + jwC_g \qquad \text{......(1)},$$

$$\partial I_a/\partial V = \partial I/\partial V + jw\,(C_{fa} + C_{ga}) = \partial I/\partial V + jwC_a \qquad \text{......(2)},$$

$$\partial I_a/\partial v = \partial I/\partial v - jwC_{ga} \qquad \text{......(3)},$$

where C_{fg}, C_{ga}, C_{fa} are the inter-electrode capacities of the valve as shown in Fig. 1. C_g may be termed the grid circuit capacity, or input capacity of the valve, and C_a its output capacity, or anode circuit capacity.

Thus each of the above admittances contains a conductance term and a quadrature term. The measurement of the input admittance $\partial i_g/\partial v$ has been described in a previous paper*. The present paper is concerned with $\partial I_a/\partial V$ and $\partial I/\partial v$.

* Hartshorn, *Proc. Phys. Soc.* **39**, 108 (1927).

§2. THE MEASUREMENT OF ANODE CIRCUIT IMPEDANCE

The application of the Wheatstone network to the measurement of the anode circuit impedance of a valve is fairly straightforward. Circuits for comparatively rough measurements of anode circuit resistance have been described by Barkhausen* and Bagally†, but for precision work and especially if the actual impedance or admittance is required, and not merely the resistance, a number of refinements are necessary. The actual arrangement used by the author for accurate measurements is shown in Fig. 2. The bridge proper is the network $DBCF$, the valve being connected in the arm FD by means of the filament and anode. The other three arms consist of non-inductive resistances R_1, R_2, and R_3, the last being shunted
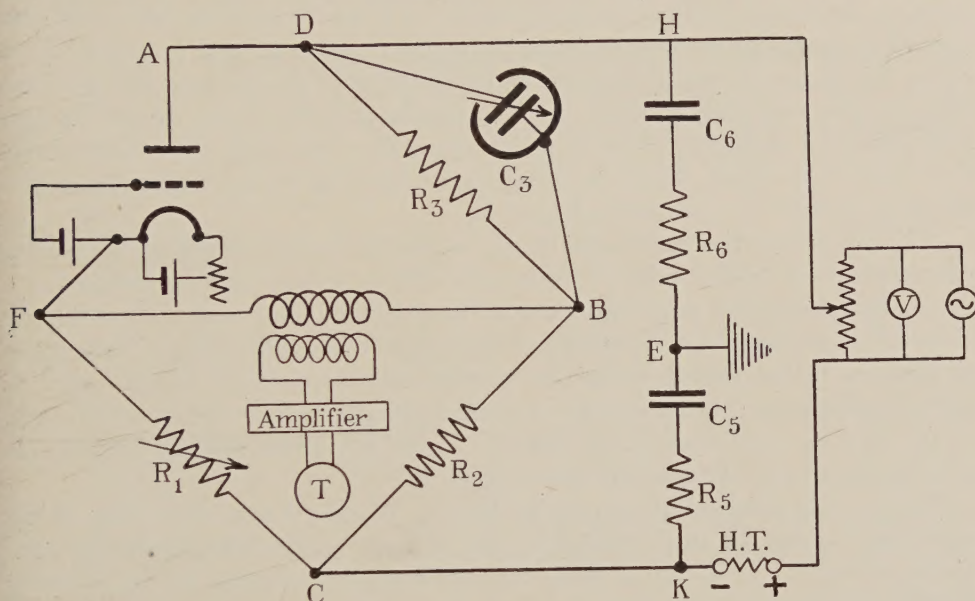

Fig. 2. Bridge for the measurement of anode circuit impedance.

by a small shielded variable air condenser C_3. A potential difference of telephonic frequency, which could be varied in magnitude from 0·2 volt to 20 volts, was applied to the points D and C, and a detector of alternating current consisting of a telephone receiver, with or without an amplifier, was applied to the points F and B.

The current flowing in the arm FD of the bridge is I_a, the anode current, and the voltage across it is V, the anode voltage. If δI_a and δV are the amplitudes of the alternating components of these quantities, then the effective impedance of the arm FD is $\delta V/\delta I_a$, and when the detector FB is silent, i.e., the bridge is balanced for the alternating components of current and voltage, it is evident that the equation for balance is

$$\frac{\delta I_a}{\delta V} = \frac{R_2}{R_1}\left[\frac{1}{R_3} + jwC_3\right] \qquad \ldots\ldots(4).$$

* Barkhausen, *Jahrbuch der drahtlosen Teleg.* **14**, 36 (1919).
† Bagally, *Journ. Sci. Instr.* **4**, 46 (1926).

Since the grid voltage v is maintained constant, if the applied voltage be reduced until the bridge readings become independent of the applied voltage, they then give the value of $\partial I_a/\partial V$ the anode circuit admittance. The two components of this are given by (see equation (2)):

$$\frac{\partial I}{\partial V} = \frac{R_2}{R_1} \cdot \frac{1}{R_3} \qquad \dots\dots(5),$$

$$jwC_a = jwC_3 . R_2/R_1 \qquad \dots\dots(6).$$

For accuracy the choice of suitable values for the various bridge components is important. Those used were $R_3 = 10,000$ ohms, $R_2 = 100$ or 1000 ohms depending on the valve to be tested. R_1 was a resistance box giving 1000 ohms in steps of $0 \cdot 1$ ohm, and C_3 had a maximum value of about $200 \mu\mu\mathrm{F}$. These resistances were completely shielded, the shields being connected to earth. A Wagner* earthing device HK was applied to the bridge, so as to maintain the points F and B at earth potential. Thus errors due to the comparatively large earth capacities of the filament and grid bias batteries are completely avoided. To increase the stability of the circuit the arm HK should be preferably of low impedance, say $R_5 = 100$ ohms, $R_6 = 1000$ ohms, $C_5 = 1 \mu\mathrm{F}$, $C_6 = 10 \mu\mathrm{F}$. These condensers are to prevent the h.t. battery from discharging through the resistances R_5 and R_6.

The h.t. battery lies outside the bridge, so that its resistance and earth capacity do not affect the balance point. Two sources of error remain to be considered:

(a) Stray magnetic coupling between the detector circuit and the a.c. supply circuit. This is allowed for by including reversing switches in the detector and supply circuits, and taking readings for the four possible relative positions of these switches, and averaging them. The total deviation of any reading from the mean is usually small.

(b) Residual inductance in the coils R_1, R_2 and R_3. These residual inductances were measured and suitable corrections applied.

Balance was obtained by varying R_1 and C_3. When the minimum value of the condenser C_3 was not small enough to give balance, another condenser was connected across R_2. It is easy to show that the complete equation for the capacity C_a takes the form

$$C_a = \frac{R_2}{R_1} C_3 - \frac{R_2{}^2}{R_1 R_3} . C_2 + \frac{R_2}{R_1 R_3} (\tau_2 - \tau_1 - \tau_3) \qquad \dots\dots(7),$$

τ_1, τ_2 and τ_3 being the time constants, or quotients of residual inductance and resistance, of the coils R_1, R_2 and R_3 respectively.

Results

Measurements of anode circuit impedance were made by this method on four typical valves. The results, which are given in Table 1, are expressed by stating the equivalent anode circuit capacity C_a, and R_a, the reciprocal of the anode circuit conductance $\partial I/\partial V$ as defined by (2). This may be defined as the anode circuit resistance†.

* K. W. Wagner, E.T.Z. 40, 1001 (1911).

† Sometimes $\partial V/\partial I$ is regarded as the anode circuit resistance, V being considered as a function of the two currents I and i. This gives a value slightly different from that given by the above definition, but the difference is of no great practical importance. See for example Nichols, Phys. Rev. 13, 411 (1919).

The results exhibit a number of interesting features. Measurements were made at frequencies of 1000, 2000, and 4000 cycles per second, but the anode circuit resistance was practically the same at all these frequencies. The actual changes shown in Table 1 were found to be due, not to the change of frequency, but either to a change in the thermionic emission of the filament with time (such changes were always very rapid when the filament current was first switched on, but gradually became slower) or to a change in the value of $\delta V/\delta I_a$ owing to the curvature of the characteristic curve of the valve. This curvature was appreciable in every case. It caused the first harmonic to be audible in the telephone even when the bridge was perfectly balanced for the fundamental, whereas if a wire wound resistance were substituted for the valve, this harmonic disappeared.

Table 1. The anode circuit resistance and capacity of various valves

Valve	Frequency, cycles per sec.	R_a (ohms)	C_a filament emitting $(\mu\mu F)$	C_a filament cold $(\mu\mu F)$
P.M. 1 A Anode voltage: 100 Fil. voltage: 2 Grid bias: zero	1000 2000 4000	77,260 77,340 77,300	17.2 16.4 16.0	13.2 12.7 12.5
D.E.R. Anode voltage: 80 Fil. voltage: 2 Grid bias: − 2	1000 2000 4000 6000	19,090 19,120 19,180 19,190	16.5 14.5 14 13.5	8.6 8.3 7.8 8
P.M. 252 Anode voltage: 100 Fil. voltage: 2 Grid bias: − 10	1000 2000 4000	3,935 3,934 3,932	21.5 18 16	14.0 13.8 13.8
R. Anode voltage: 100 Fil. voltage: 4 Grid bias: zero	1000 2000 4000	27,700 27,700 27,600	35.0 27.5 24.5	20.8 18.5 17

The equivalent anode circuit capacity C_a is by no means independent of the frequency. In the case of the P.M. 252 valve, C_a at 1000 $\sim$ is 30 per cent. greater than it is at 4000 $\sim$. The change is rather less in other cases, but in every case the equivalent anode circuit capacity decreases with increase of frequency.

The last column of Table 1 gives the values of the anode circuit capacity when the filament current was switched off. It consists of the sum of the two capacities C_{fa} and C_{ga} together with the capacities of connecting leads, etc. It was measured on the same bridge, without in any way disturbing the leads or batteries. On switching off the filament current a standard resistance of approximately the same value as the anode circuit resistance of the valve was connected across FD (Fig. 2), so that when balance was again obtained the conditions were as nearly as possible the same as in the first instance. It is to be noted that, in every case, the effective anode circuit capacity, when the thermionic current is flowing, is greater than the capacity of the same circuit when the filament is cold. Moreover, this capacity with

the filament cold is sometimes very nearly independent of the frequency, whereas the capacity with the filament glowing always varies with the frequency.

The explanation of these facts is to be found in the presence of the space charge between the filament and the anode. The free electrons constituting this charge endow this space with a certain conductivity which is not uniform, but is greatest near the surface of the filament, and decreases as we approach the anode. In other words, the filament regarded as a conducting electrode is increased in size by the space charge (its boundary becoming indefinite) and the filament-anode capacity is therefore increased.

Approaching the matter from another point of view, if ϵ is the dielectric constant of the medium at any point between filament and anode, and σ is the conductivity at this point due to the free electrons, then it is evident that the quotient ϵ/σ will vary from point to point as we pass from filament to anode. The space, therefore, possesses the properties postulated by Maxwell* in his theory of dielectric absorption, and the mathematical development of this theory shows that the following phenomena are a necessary consequence of this property:

(*a*) The capacity between the filament and anode must be greater than their "geometrical mutual capacity," which is practically the same as the capacity when the filament is cold.

(*b*) The increase in capacity will vary with the frequency, being greatest at low frequencies, and gradually approaching zero at very high frequencies.

(*c*) The conductance between filament and anode when alternating voltage is applied will be greater than the conductance with constant applied voltage, i.e. the "static" value. The a.c. conductance will increase as the frequency increases from zero, approaching a constant value at high frequencies.

This provides a complete explanation of the capacity values given above. The change in conductance with frequency was not actually observed. It is evidently very small in amount, and was masked by variations due to other causes, but it is interesting to note that the resistance of a valve as determined from the characteristic curves must be slightly different from the dynamic value, or value determined by a.c. methods, the a.c. resistance being the smaller.

The alternating current conductance due to the space charge is best measured by a measurement of input impedance, in which case this conductance forms a large part of the total conductance, and is therefore not likely to be masked by other effects. The results given in a previous paper† show the effect quite clearly, though their significance was not fully realised at the time. The input capacity of an R. valve was found to be $13 \cdot 0 \mu\mu F$, with a power factor of $0 \cdot 21$ at 1000 cycles per second when the filament was cold. On the application of full voltage to the filament, the capacity rose to $14 \cdot 6 \mu\mu F$, and the power factor became as high as $0 \cdot 38$. The increase of power factor is attributed to a grid circuit conductance ($0 \cdot 015$ micro-mho), due to the space charge. This conductance existed even when negative grid bias was used. An important conclusion is that the inter-electrode capacities of a valve

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* Maxwell, *Electricity and Magnetism*, 1, 328; Hartshorn, *I.E.E. Journal*, 64, 1152 (1926).
† Hartshorn, *Proc. Phys. Soc.* 39, 114 (1927).

must be regarded as having comparatively high power factors at audio frequencies when the valve is working, even if measurements made when the filament is cold give very low power factors. The space charge may give rise to alternating current conductance even under conditions under which no d.c. conductance is detectable. The power factor due to the space charge will be greatest at low frequencies, gradually decreasing as the frequency is raised, as in the case of a non-homogeneous dielectric*.

§3. THE MEASUREMENT OF MUTUAL CONDUCTANCE

Fig. 3 shows the Wheatstone Bridge network arranged for the measurement of the mutual conductance of a valve. R_1, R_2, R_3 are non-inductive resistances as in the previous case and C_3 is a small variable air condenser. The valve is connected in the fourth arm by means of the grid and filament, the anode being connected
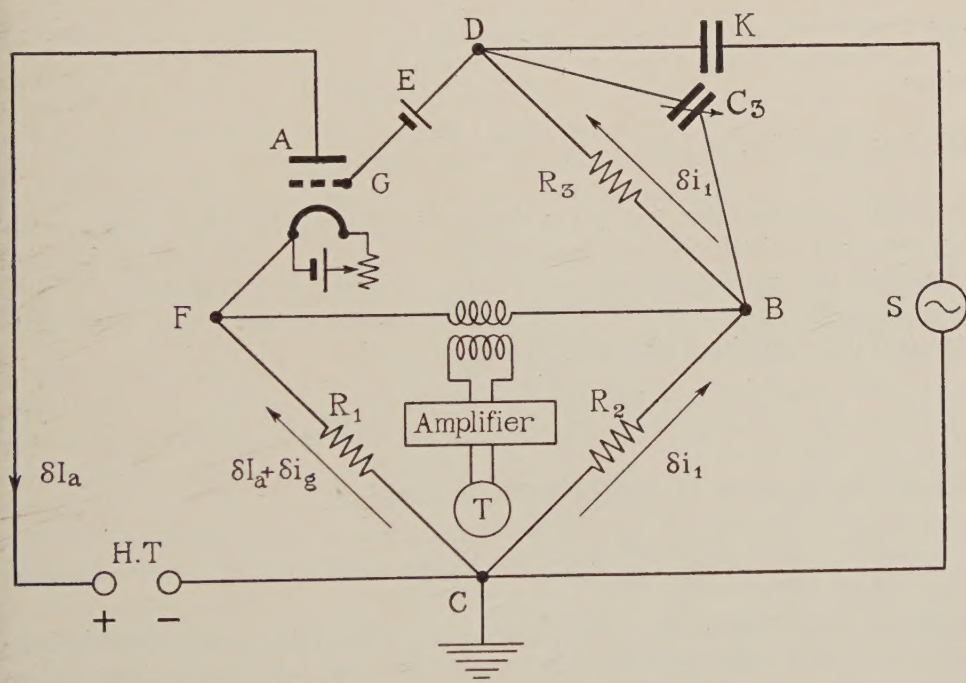

Fig. 3. Bridge for the measurement of mutual admittance.

through the h.t. battery to the point C. The source of telephonic frequency current and the detector of balance are the same as in the previous case, except that the applied voltage may be smaller, say 0·02 volt, if an amplifier is used in the detector circuit. The mean potential of the grid with respect to the filament is adjusted to any desired value by placing a "grid bias" battery E in series with the grid as shown in the diagram.

* Hartshorn, *I.E.E. Journal*, **64**, 1152 (1926).

In carrying out the measurements described later, bridge components of the following values were used, $R_1 = R_2 = 100$ ohms, $R_3 =$ a resistance box giving 10,000 ohms total, continuously variable, $C_3 = 200\mu\mu F$ maximum. The bridge was balanced with respect to alternating current by varying R_3 and C_3 until the telephone became silent.

Since the telephone detector responds only to variations of current or voltage, in deriving the equations of balance we need only consider the alternating components δi_g, δI_a, δv, and δV, of the grid current, anode current, grid voltage, and anode voltage respectively. When the bridge is balanced, there is no alternating current in FB. Thus the current flowing from C to F will be equal to the sum of those flowing from F to G, and from F to A, i.e. $\delta i_g + \delta I_a$. Let δi_1 be the alternating current in the arms CB and BD. Then remembering that the alternating p.d. across the arm FD is δv, it is evident that the conditions of balance are

$$R_1(\delta i_g + \delta I_a) = R_2 . \delta i_1 \qquad \ldots\ldots(8)$$

and

$$\delta v = Z_3 . \delta i_1 \qquad \ldots\ldots(9),$$

where

$$1/Z_3 = 1/R_3 + jwC_3 \qquad \ldots\ldots(10).$$

Dividing (8) by (9), we have

$$\frac{\delta i_g}{\delta v} + \frac{\delta I_a}{\delta v} = \frac{R_2}{R_1} . \frac{1}{Z_3} \qquad \ldots\ldots(11).$$

It should be noticed that the load in the anode circuit of the valve consists of the h.t. battery of resistance R_b say, in series with R_1. If $R_1 + R_b$ be made small compared with the anode circuit resistance of the valve, the anode potential V will be approximately constant, and if in addition the applied voltage is very small equation (11) may be written with sufficient approximation

$$\frac{\partial i_g}{\partial v} + \frac{\partial I_a}{\partial v} = \frac{R_2}{R_1} . \frac{1}{Z_3} = \frac{R_2}{R_1}\left[\frac{1}{R_3} + jwC_3\right].$$

Substituting equations (1) and (3) in this, we find

$$\frac{\partial i}{\partial v} + \frac{\partial I}{\partial v} + jwC_{fg} = \frac{R_2}{R_1}\left[\frac{1}{R_3} + jwC_3\right].$$

Equating the real terms of this expression and remembering that the grid conductance $\partial i/\partial v$ is sensibly zero in most practical cases, we have the working equation for mutual conductance measurements

$$\frac{\partial I}{\partial v} = \frac{R_2}{R_1} . \frac{1}{R_3} \qquad \ldots\ldots(12).$$

In precision measurements it is sometimes necessary to apply a correction for the small variation of anode voltage. This variation is given by

$$\delta I_a R_b + (\delta I_a + \delta i_g) R_1 \quad \text{or} \quad (R_1 + R_b)\delta I_a,$$

the remaining term being negligible.

Now
$$\delta I_a = \frac{\partial I_a}{\partial v} . \delta v + \frac{\partial I_a}{\partial V} \delta V \quad \text{(approximately)}$$

or
$$\delta I_a \doteqdot \frac{\partial I_a}{\partial v} . \delta v + \frac{R_1 + R_b}{R_a} \delta I_a$$

since
$$\frac{\partial I_a}{\partial V} \doteqdot \frac{\partial I}{\partial V} = \frac{1}{R_a},$$

$$\therefore \frac{\partial I_a}{\partial v} = \frac{\delta I_a}{\delta v} \left[1 - \frac{R_1 + R_b}{R_a} \right] \qquad \ldots\ldots(13).$$

The term in brackets is therefore the required correction factor, and equation (12) is replaced by the more accurate equation

$$\frac{\partial I}{\partial v} = \frac{R_2}{R_1} . \frac{1}{R_3} \left[1 - \frac{R_1 + R_b}{R_a} \right] \qquad \ldots\ldots(14).$$

In order to obtain the actual value of the mean potential of the grid with respect to that of the "zero" end of the filament, i.e. the "grid bias," the values of the constant components of the currents and voltages must be considered. The constant component of the anode current I flows from C to F, partly by way of R_1, and partly by way of the path CBF. Thus there is a constant current in the detector arm BF, and the bridge is therefore not balanced as regards the constant components of current and voltage. A condenser K is placed in series with the source of alternating current S. This prevents any constant current flowing from D to B via S, and at the same time allows the passage of alternating current. A condenser of $0.5\,\mu F$ was used for K, but it could be much less. Owing to the very high resistance of the grid circuit of the valve, the constant current in the bridge arms DF and DB is very small. The mean potential of D is therefore practically the same as that of B, and if E is the e.m.f. of the battery E, and e is the p.d. between B and F, the "grid bias" or the p.d. between G and F is $(E + e)$. The value of e may easily be calculated, if the total anode current I and also the d.c. resistances of FB, R_1, and R_2 are known, but it is more convenient to use a transformer with a low resistance primary winding FB to couple the bridge to the amplifier. In this way e can be made negligibly small, so that the grid bias is given simply by E. Since the resistance of the grid bias battery is included in the measurements, it should be as small as possible, i.e. an accumulator should be used, but in doubtful cases the resistance of this battery can always be eliminated by taking an additional reading with the battery transferred to the arm DB. The grid bias is unaltered, and the mean of the two readings will give the true value of the mutual conductance. It is also possible to work with the grid bias battery in the detector arm FB, and so remove it from the bridge arms proper. This arrangement was used for some measurements, but it is considered to be not quite so convenient as the one shown in Fig. 3.

When the quadrature component of the admittance is not required, it is not necessary to use a Wagner earth. Sufficient accuracy is obtained by earthing the

point C, at the same time making the resistances R_1 and R_2 as small as possible (100 ohms or even 10 ohms). This ensures that the detector arm FB is not much above earth potential. The same applies to the measurement of anode circuit resistance.

The thermionic current I is usually expressed as a function of the single variable $(V + \mu v + c)$, where μ is the "voltage factor" of the valve:

$$I = F(V + \mu v + c) \qquad \ldots\ldots(15).$$

It follows that

$$\partial I/\partial v = \mu . \partial I/\partial V \qquad \ldots\ldots(16).$$

Thus the product of the anode circuit resistance R_a, and mutual conductance G_m, of a valve at any operating point (V, v) gives the value of μ.

With the simple earthing arrangement, values of both R_a and G_m can be obtained very quickly over the whole working range of grid bias.

Results

Measurements were made in this way on a few typical valves. The values of R_a and G_m are given in the third and fourth columns of Table 2. Their product (μ) is given in the last column. It is to be noted that although the values of R_a and G_m vary considerably with variations of grid bias, the product is very nearly constant. The variations in the values of μ in Table 2 show with what accuracy this may be regarded as a true constant for any one valve.

With regard to the accuracy of the measurements, a number of factors have to be considered besides mere sensitivity. The sensitivity was in all cases greater than 0·1 per cent., using an input p.d. of 0·02 volt for the mutual conductance and 0·2 volt for the anode circuit resistance measurements, but the properties of the valves were found to vary appreciably with time of application of the various voltages. In the case of S.P. 18 R.R. valve, a change of about 10 per cent. was liable to occur in 10 minutes, but after a time the valve appeared to settle down, although irregular fluctuations of 2 or 3 per cent. always occurred. The other valves were much more stable, and the values of G_m and R_a are probably trustworthy to about 0·2 per cent. in most cases.

An examination of the results for R_a and G_m for various values of grid bias shows that the characteristic curves of the valves cannot be regarded as really straight even over the usual working portion. Very considerable changes of slope occur from point to point, and it is evident that no single value of anode circuit resistance can be assigned to any valve with an accuracy greater than 10 per cent. unless all the operating voltages are specified.

The values of $1/R_a$ and G_m may be regarded as the slopes of tangents to the characteristic curves at the operating point, and in order to determine these it is, strictly speaking, necessary to use indefinitely small applied voltages. The bridges actually measure the slopes of two chords instead of tangents, since the applied voltage is necessarily finite. When using applied voltages of the order 0·1, errors due to this are usually negligible, e.g. doubling the applied voltage caused no

appreciable change of bridge reading in general, but when the operating point (V, v) is near the bend of the characteristic curve, particular attention must be paid to this point, especially when μ is to be determined from the product $G_m R_a$. A consideration of equation (15) will show that μ will be correctly given by the effective values of G_m and R_a even when the applied alternating voltage (δV or δv) is not very small, provided that $\delta V/\delta v = \mu$. It was on this account that the applied voltage for the G_m measurements was roughly one-tenth of that used for the R_a measurements.

Table 2. Measured values of mutual conductance (G_m), anode circuit resistance (R_a), and voltage factor (μ) of typical valves

Valve	Grid bias voltage	R_a (ohms)	Mutual conductance (G_m) micro-mhos	Voltage factor (μ)
P.M. 1 L.F. Fil. voltage: 2·0 Anode voltage: 100	+ 4	10,370	797	8·2₆
	+ 2	10,400	828	8·6₁
	0	10,630	820	8·7₂
	− 2	11,280	770	8·6₉
	− 4	12,410	701	8·7₀
	− 6	14,720	593	8·7₃
P.M. 1 H.F. Fil. voltage: 2·0 Anode voltage: 80	+ 4	17,020	992	16·8₉
	+ 2	18,940	910	17·2₃
	0	22,520	762	17·1₆
	− 2	32,810	524	17·2₀
	− 4	151,200	108.₀	16·3₃
D.E.H. Fil. voltage: 2·0 Anode voltage: 150	+ 2	54,630	757	41·3
	0	71,100	575	40.9
	− 2	156,700	248.₄	38·9
D.E.R. Fil. voltage: 2·0 Anode voltage: 80	+ 4	20,200	539	10·9
	+ 2	21,680	514	11·1
	0	24,310	451	11·0
	− 2	28,830	355	10·2
	− 4	37,480	253.₉	9·5
S.P. 18 R.R. Fil. voltage: 1·6 Anode voltage: 100	+ 4	3,730	1727	6·4₄
	+ 2	3,960	1800	7·1₂
	0	4,055	1669	6·7₆
	− 2	4,440	1550	6·8₉
	− 4	4,617	1386	6·4₀
	− 6	5,100	1191	6·0₈
R.C. 2 Fil. voltage: 2·0 Anode voltage: 100	− 1	338,000	135	45·6
	− 2	512,000	85	43·5

DISCUSSION

Dr J. H. VINCENT: These experiments of Mr Hartshorn are of great use as well as elegance. The results are fortunately not revolutionary. In particular it is a comfort to be assured that the voltage factor measured by his methods remains sensibly constant.

Mr A. G. WARREN: In view of increase of inter-electrode capacity due to space charge, it would appear that values C_a, C_g etc. would be functions of grid voltage. Has the author noted any greater increase in filament/anode capacity for a gettered valve than for one which is not gettered? One would anticipate that the presence of a conducting internal coating on the glass, which naturally tends to assume cathode potential when the filament is lighted, would increase the effective filament/anode capacity.

Mr W. E. BENHAM: I should like to endorse the last speaker's remarks concerning the variation of capacity with voltage. It would appear that there is a term of the form $v\, \partial C/\partial v$ missing from equations (1) to (3). As a result of some experiments on valve capacities I came to the conclusion that the self capacity of a triode could be expressed approximately by the equation

$$C = C_0\,(1 + A/R_a),$$

where A is constant for a given anode voltage (120 volts) and equal to about 6×10^3, R_a being the anode slope resistance in ohms. The measurements were made on Osram D.E.P. 215 and D.E.H.L. 210 type valves at a frequency as high as 5×10^6 cycles per second, using a resonance method incapable of high precision.

The author's conclusions as to the increase of valve capacity (at telephonic frequencies) due to space-charge are of great interest, and are in qualitative agreement with a calculation giving $C = \frac{1}{3} C_0$ for zero frequency. The author's experimental method should be capable of yielding accurate results over a wide range of frequencies, but until a correction is made I think his figures for the output capacity and its frequency-variation must be accepted with reserve. Another consideration presenting difficulties is the influence of traces of ionization.

AUTHOR'S reply: I agree with Mr Warren that the effective values of the valve capacities are functions of the grid voltage, v. Indeed, I consider that one of the most important conclusions to be drawn from my investigation is that the effective inter-electrode capacities of valves are not simply the same as the corresponding "static" capacities, but that they vary with the position on the characteristic curve of the operating point (I, v), and therefore the resistance of the valve, and also with the frequency. My results show the kind of variation and its order of magnitude. I have not made comparative measurements on valves gettered and not gettered.

Mr Benham's suggestion that there is a term of the form $v\,\partial C/\partial v$ missing from equations (1) to (3) is probably due to a misunderstanding of the nature of these equations. They must be regarded as *defining* the conductances and capacities measured by the bridge circuits. In each case they give the two quadrature components of a vector, and indicate their relationship to the three valve electrodes. Each equation applies at any one operating point (I, v) and the values of the terms are, of course, quite different at different operating points. One must realize that the electrical path between any two valve electrodes is neither a conductor nor an insulator, and that the terms conductance and capacity can only be applied

in accordance with some arbitrary convention. Equations (1) to (3) express the convention used most widely in the study of imperfect dielectrics, and in electrical measurements generally. The introduction of another term would imply a different convention, and one inconsistent with general practice. On this understanding, there can be no question of correcting the results of the bridge measurements, and the presence of traces of ionization or any deposit on the "pinch" of the valve will cause no difficulty. They will affect the value of the capacity, but the bridge will measure the actual capacity under the working conditions.

I have made calculations of C/C_0 at zero frequency, and agree with Mr Benham's value of $\frac{4}{3}$ for the case of plane electrodes, and with Langmuir's theoretical formula for the thermionic current. This is, however, not a very practical case. My calculations show that the value of C/C_0 depends greatly on the form of the filament, and on the actual formula used for the thermionic current. Taking various cases, values ranging from 1 to 2 or even greater can be obtained for C/C_0 at zero frequency. In this connection, I presume that the constant A in Mr Benham's equation $C/C_0 = 1 + A/R_a$ refers only to electrodes of one particular form. However, bearing in mind that the effect must be smaller at the higher frequencies, there does not seem much possibility of getting trustworthy results at radio frequencies, unless a high precision method be used, and in particular unless all stray earth-capacity effects be eliminated. The method I have described could, of course, be used at high frequencies, provided that suitable bridge components were used and that everything were properly screened. The practical difficulties would be considerable, but not insuperable.

A METHOD FOR THE DETERMINATION OF THE EQUIVALENT RESISTANCE OF AIR-CONDENSERS AT HIGH FREQUENCIES

By G. W. SUTTON, B.Sc.

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ABSTRACT. The losses in air-condensers are divided into two portions, (1) those due to leakage through the solid dielectric, and (2) those due to terminal and plate resistance. A method is developed for measuring each, under conditions such that the other is negligibly small. The limits of the errors to which the methods are liable are discussed and some results of practical measurements are quoted.

THE losses in air-condensers such as are used in medium and high-frequency a.c. measurements may be safely assumed to arise from two sources, (1) Losses in the solid dielectric essential to the mechanical construction, and (2) The resistance of the leads from the terminals to the plates and of the plates themselves.

§ 1. LOSSES IN THE SOLID DIELECTRIC

The paths along which the first of these losses occur are limited to portions of solid dielectric between the terminals, pillars, etc., and the partial capacities due to flux along these paths form only a small portion of the total effective capacity, even at the minimum setting of a variable air-condenser. Consequently a high non-inductive resistance in parallel with a perfect condenser may, so far as such losses are concerned, be safely considered an equivalent circuit at any given frequency. The measurement with fair accuracy of the equivalent parallel resistance offers little difficulty.

A suitable circuit arrangement is shown in Fig. 1, in which C_1 is the condenser under examination and C_2 a second variable condenser of good construction and of about $750\mu\mu\mathrm{F}$ capacity. C_1 is set at its minimum reading, which will usually be of the order of $50\mu\mu\mathrm{F}$ or less. For any given frequency the coil L should be chosen so that it is necessary to set C_2 at about $500\mu\mu\mathrm{F}$ for resonance.

On switching C_1 out of circuit by the small link S the alteration of the C_2 necessary to restore resonance will be only 10 per cent. or so of its actual value, and it may be assumed that the change of resistance of C_2 due to the alteration is negligibly small. The lead resistance of C_1 when set at its minimum value will, in a good condenser, be quite small compared with the series resistance equivalent to its solid insulation losses. Consequently the change of resistance of the tuned circuit on removing S may be assumed to be equivalent to these insulation losses

and may be readily measured with the aid of the voltmeter and the resistance substitution method. The results of measurements which show the above assumptions to be permissible are given later. (See Tables 1 and 4.)

§ 2. RESISTANCE OF THE LEADS AND PLATES

The similarity of the distribution of the lines of current-flow between two electrodes immersed in a conducting solution and of the lines of electric force between two insulated and charged surfaces of the same area, shape and relative position, has frequently been used to estimate the capacity between the surfaces*. The principle has also been used in the investigation of the electro-magnetic field of machines. In the present case it has been employed to enable a measurement of high-frequency resistance of the leads and plates of an air-condenser to be obtained. The condenser is immersed in a suitable electrolyte and the resistance from terminal to terminal is measured. Allowance is then made for the electrolytic resistance, which is usually a small proportion of the total, by determining the specific resistance of the electrolyte and employing the relationship† $R = K\rho/4\pi C$, where R is the electrolyte resistance (in absolute units), K the constant of the dielectric, C the capacity of the condenser when used normally, and ρ the specific resistance of the electrolyte.

An alternative method of measurement

Alternatively, the condenser is immersed in several solutions, having different values of ρ, and the resistance measured from terminal to terminal is plotted to a base of ρ. On extrapolating to the axis where $\rho = 0$, the intercept gives the required terminal and plate resistance.

Errors to which the method is liable

The method is liable to two important sources of error, but it is possible to avoid them and to obtain a very fair accuracy of result. The first of these is the possibility that at high frequencies the eddy-currents in the electrolyte may modify the lines of current-flow. A theoretical estimate of the limit of this error in any particular case may, however, be made. Let it be assumed that the overall diameter of the condenser is 20 cm. and that the frequency of measurement is $10^6 \sim$. The electrolyte may be considered to be a circular conductor of this diameter, and if the calculated high-frequency resistance of such a conductor does not differ appreciably from the D.C. value it is probably safe to assume that the lines of the current-flow between the plates of the condenser when immersed in the electrolyte are an accurate replica of the lines of force between the plates when the condenser is used normally. If this is so the current distribution and consequently the "skin-effect" in the plates and leads will be identical in the two cases. A suitable

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* A. E. Kenelly and S. E. Whiting, *Elec. World*, **48**, 1238 (1906).

† *Loc. cit.*

value of ρ is 50 ohms/cm./cm.[2] Actually electrolytes having values between 20 and 300 ohms have been employed.

The value of z in the formula $z - \pi d \sqrt{2f \rho}$ is 0·4 and from the tables* F is 0·000, consequently $R_{hf} = R_{dc} \cdot (1 - F) - R_{dc} \cdot 1\cdot000$. Thus it should be perfectly safe to employ an electrolyte of the above resistance in any ordinary condenser at $f = 10^6 \sim$.

The second source of error mentioned above is that the boundary surface of the electrolyte in which the plates are immersed may not coincide with that of the dielectric when the condenser is used normally. This error may be reduced to a negligible minimum for all but the lowest settings of a shielded variable air-condenser by suitably shaping the vessel in which the plates are immersed. At the lowest readings, however, the error becomes appreciable.

The readings given in Table 1 illustrate the limits which are imposed by the two errors just discussed:

Table 1

Frequency $10^6 \sim$.   Variable air-condenser No. 1950.
Electrolyte: common salt dissolved in tap-water.

Con-denser setting ($\mu\mu$F)	Approximate strength of solution											
	N/2			N/5			N/10			N/20		
	Value of ρ (ohms)											
	33·5			65·7			111			182		
	Resistance (ohms)											
	A	B	C	A	B	C	A	B	C	A	B	C
1144	0·15	0·0026	0·14	0·13	0·0051	0·12₅	0·13	0·0086	0·12	0·13	0·0141	0·1
889	0·14	0·0033	0·14	0·13	0·0065	0·12₅	0·13	0·0111	0·12	0·13	0·0181	0·1
693	0·14	0·0043	0·14	0·14	0·0084	0·13	0·14	0·0142	0·12₅	0·14₅	0·0232	0·1
443	0·14₅	0·0067	0·14	0·15	0·0131	0·14	0·16	0·0222	0·14	0·17	0·0363	0·1
325	0·15	0·0091	0·15	0·16	0·0179	0·15	0·17	0·0303	0·14	0·19	0·0495	0·1
208	0·16	0·0142	0·15	0·18	0·028	0·15	0·21	0·0473	0·16	0·25	0·0773	0·1
92	0·17	0·0322	0·14	0·22	0·063	0·16	0·29	0·107	0·18	0·30	0·174	0·2
57	0·17	0·052	0·12	0·23₅	0·102	0·13	0·34	0·172	0·17	0·50	0·281	0·3

Columns A give the total measured resistance, employing the resistance substitution method.
Columns B give the resistance of the electrolyte calculated from the measured values of ρ.
Columns C give the difference between A and B; i.e. the lead and plate resistance.

So far as the eddy-current error is concerned, it would appear from the results that its effect is just appreciable when the N/2 solution is used, but negligible with the more dilute solutions.

The error due to the impossibility of completely replacing the dielectric with electrolyte is apparent in the increase in columns C with decreasing condenser setting, particularly with solution N/20. The total measured resistance would, of course, be lower in these cases if complete substitution were possible. The discrepancy is greater in the readings just given than in those subsequently obtained, as insufficient care was taken in filling the condenser-casing with the solution.

* S. Butterworth, "The Effective Resistance of Inductance Coils," *Exp. Wireless*, **3**, 207 (1926).

Subsequently it was found that the error was negligible down to a setting of about $200\,\mu\mu F$.

The important point brought out by these readings is that the electrolyte, although having a specific resistance high enough to ensure freedom from eddy-current error, may yet have an effective resistance, when in the condenser, low enough to enable measurement of the plate and lead resistance to be made with reasonable accuracy.

An experimental example

As an example of the usefulness of this method the following experiment may be quoted.

An air-condenser which was being used at very high frequency was suspected of having an unduly high resistance. Upon immersing it in a suitable container filled with electrolyte the following results were obtained:

Frequency $5 \times 10^6 \sim$
Solution strength $N/3$ $N/6$ $N/10$
Resistance* (ohms)   2·04   2·11   1·73

Upon repetition the readings were found to be very irregular, ranging from 1·5 to 2·5 ohms. The condenser was therefore taken to pieces and all surfaces between which electrical contact was important were carefully cleaned. When the instrument was being reassembled, all nuts were screwed up tight and heavy gauge copper wires were carefully soldered from plate to plate on both the fixed and moving systems.

The readings were then:

Frequency $5 \times 10^6 \sim$
Solution strength $N/3$ $N/6$ $N/10$
Resistance (ohms)   0·27   0·27   0·28

As a final test of the consistency of the results obtainable, the condenser previously used (No. 1950) was taken to pieces, cleaned and rebuilt, and the measuring circuit was arranged to give as great a sensitivity as possible.

The readings then obtained are shown in Tables 2 and 3.

Table 2

Frequency $10^6 \sim$.   Condenser No. 1950.

Strength of solution ...	$N/2$	$N/5$	$N/10$	$N/20$
Value of ρ (ohms) ...	34·2	64·5	108	183
Condenser setting ($\mu\mu F$)	Resistance* (ohms)			
1153	0·15$_4$	0·13$_3$	0·12$_6$	0·12$_1$
892	0·14$_1$	0·12$_8$	0·11$_9$	0·11$_3$
688	0·13$_6$	0·12$_6$	0·11$_2$	0·11$_4$
439	0·12$_9$	0·12$_4$	0·11$_0$	0·11$_1$
319	0·12$_8$	0·12$_3$	0·10$_9$	0·10$_4$
208	0·12$_2$	0·11$_9$	0·11$_2$	0·11$_0$

* The electrolyte resistance was negligibly small.

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Table 3

Frequency $0.5 \times 10^6 \sim$.   Condenser No. 1950.

Strength of solution ...	$N/2$	$N/5$	$N/10$	$N/20$
Value of ρ (ohms) ...	34·2	64·5	108	183
Condenser setting ($\mu\mu$F)	Resistance* (ohms)			
1153	0.08_1	0.07_4	0.07_4	0.06_9
892	0.08_0	0.07_3	0.07_1	0.06_7
688	0.07_9	0.07_1	0.06_9	0.06_5
439	0.07_6	0.06_8	0.06_7	0.06_2
319	0.07_3	0.06_8	0.06_6	0.06_0
208	0.07_4	0.07_4	0.06_8	0.06_6

* The electrolyte resistance has been subtracted throughout.

The results in Table 2 are somewhat high for $N/2$ and $N/5$ and in Table 3 for $N/2$ solution when compared with the remaining figures, indicating a tendency for eddy-current effects to modify the value of the resistance as measured. There is throughout a decrease of resistance as the condenser setting is decreased.

The insulation resistance of condenser No. 1950 was measured by the method indicated at the outset, with the result shown in Table 4.

Table 4. Insulation resistance of condenser No. 1950

Frequency $\sim$ Equivalent series resistance at minimum setting of $64.7\,\mu\mu$F (ohms)	0.5×10^6	10^6	2×10^6	4×10^6
	7·985	3·74	1·81	0·91

On the assumption that the parallel resistance equivalent to the insulation losses is independent of the condenser setting, which is probably justified in view of the non-distributed nature of the leakage path, the results in Tables 2 and 4 may be used to obtain the total condenser resistance at various settings.

Table 5

Frequency $10^6 \sim$.   Condenser No. 1950.

Condenser setting ($\mu\mu$F)	Insulation loss— equivalent series resistance (ohms)	Plate and lead resistance (ohms)	Total resistance (ohms)
64·7	3·74	(0·11)	(3·85)
100	1·56	(0·11)	(1·67)
208	0·36	0·11	0·47
439	0·08	0·11	0·19
688	0·04	0·11	0·15
892	0·02	0·12	0·14
1153	0.01_2	0·13	0.14_2

At these high frequencies, with the large electrode surface-area of the condenser when immersed in electrolyte, the effect of electrode capacity and its losses may be ignored. It necessitates only a very slight readjustment of the tuning of the test-circuit when the immersed condenser is switched in and out of circuit during the measurement of its resistance by the resistance substitution method.
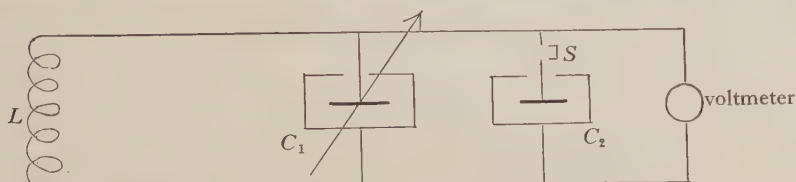

Fig. 1. Resonant circuit employed in measuring the insulation losses in C_1.

§ 3. ACKNOWLEDGMENTS.

The experimental work has been carried out at the City and Guilds (Eng.) College, where the author has had the advantage of discussing his results with Professor C. L. Fortescue, O.B.E., M.A., and Assistant-Professor E. Mallet, D.Sc.

DISCUSSION

The author, when introducing the paper, mentioned that the work had been done, for the most part, nearly two years before its publication. This explained why no reference was made to results recently published on the same problem. There are important points, both of agreement and disagreement, between the author's results and those obtained by Mr Willmotte* and Dr Dye†. In the present paper it is shown that the solid insulation losses have an equivalent series resistance very closely proportional to $1/f$. In both of the papers quoted the solid insulation is found to have a constant power-factor. On the other hand, Mr Willmotte finds that the remaining losses may be represented by a fixed series resistance, whereas the author's results indicate a rapid increase of this term with frequency. If such losses are ascribable to "skin-effect," one would certainly expect an increase with frequency.

Mr ALBERT CAMPBELL (communicated): It is difficult to interpret the expression ohms/cm./cm.² used by the author to denote units of resistivity; the use of one solidus following another is confusing. The ordinary, and to my mind the more rational, notation for such units is ohm-cm. or microhm-cm.

Mr G. L. ADDENBROOKE: I do not pretend to have had much experience with the measurement of the losses in condensers at high frequency, though I have done a good deal of work at power and telephone frequencies; nor do I feel

* "A Quick and Sensitive method of measuring Condenser Losses at Radio-Frequencies," *Journ. Sci. Instruments*, Dec. 1928.
† "Basic measurements of the Effective Resistance of Condensers at Radio Frequencies," *Proc. Phys. Soc.* Vol. **40**, Pt 5 (1928).

competent on theoretical grounds to compare any advantages there may be in the author's methods with those of other methods of obtaining the losses in air condensers and their plates. On practical grounds, however, I should have considerable hesitation in applying the method to a condenser on whose after performance I placed any serious value. Not long after the war I purchased several air-condensers with the view of seeing how far they could be used as standards, and made measurements of their dielectric losses and general characteristics. Without going into details, I may say that I found greater differences in the losses than one would have expected from a general inspection of the condensers themselves. I also found that the losses differed from day to day as the moisture in the air varied. In this connection I would point out that surface leakage with alternating currents cannot be predicted from ordinary insulation measurements. In a short paper which I read before the Society in 1912, I showed that even at ordinary frequencies the losses over the surfaces of the class of materials employed in insulating condensers may readily be a hundred times as great, and possibly more at high frequencies. In fact in their general nature they follow the characteristics of the losses through dielectrics; the losses arise, apparently, from the presence of moisture in both cases. To wet the surface of a dielectric and to anticipate that, by drying, you will in a measurable time bring it again to exactly the state in which it was before, is a very doubtful proceeding. I once or twice experienced much trouble and difficulty from moisture unexpectedly getting in between metal surfaces screwed down on to a dielectric; such moisture may be quite invisible, surface moisture being got rid of for the moment, but it will ooze out and affect the insulation in its neighbourhood for a long time. The insulation of high class air-condensers is a subject which requires much care and judgment, as the insulation of many materials deteriorates with time; and my own view is that once a good insulating surface has been obtained, it should be kept in the dark and in well-dried air to maintain it in the same condition.

Mr L. HARTSHORN: The novel application of the similarity of lines of conduction current flow in an electrolyte, and of displacement current flow in a dielectric, has possibilities in considering questions of design, but I do not think it should be regarded as a general method of test for variable air-condensers. Very few insulators, if any, can be immersed in an electrolytic solution without suffering a change, so that the losses in the solid dielectric will probably be quite different after the test from what they were before it.

Prof. F. L. HOPWOOD: I should like to call attention to the fact that the resistances of electrolytes at frequencies of the order of one million do not vary regularly with concentration when high dilutions are being used*.

The PRESIDENT: I should like to ask whether the author has studied the effect of the large dielectric constant of an electrolyte in altering the distribution of high frequency current in a condenser, and especially the distribution of the displacement current in the insulators wherein the dielectric losses presumably occur.

* See "Diathermie," by Bordier, Paris 1928, and Fabry, *Comptes Rendus* (1927).

The immersion in an electrolyte would doubtless directly affect the value of the surface resistivity of the submerged parts of the insulation. The method proposed is bold and ingenious, and I hope the author will study it further and justify it against the above and similar criticisms.

Prof. E. MALLETT: As the author mentioned in introducing the paper, when his work was done there was no direct method (apart possibly from a thermal one) of measuring the losses in an air-condenser at high frequencies, and no attempt had been made to separate the losses into those due to the insulation of the condenser and those due to the high frequency resistance of the plates. In these circumstances I think the author is to be congratulated upon his ingenious solution of a very difficult problem.

AUTHOR'S reply: In answer to Mr Campbell: The notation ohms/cm./cm.^{2} appears to me to be the briefest way of expressing the resistance of 1 cm. length of a conductor of 1 cm^2. cross-sectional area, and is to be preferred to ohm-cm. as being a trifle more definite. Engineers frequently find it convenient to express the specific resistance of copper as $\frac{1}{58}$ ohm per metre per mm.^{2}, and the above notation falls into line with this practice.

Mr Addenbrooke mentions the deleterious effect on solid insulation of immersion in electrolytes. The electrolyte finally selected for my experiments was an aqucous solution of common salt, as this happens to have no appreciable chemical action on either ebonite or the brass and copper portions of a condenser, at least during a brief immersion. It is true that moisture will, under the influence of light, form traces of sulphuric acid from the free sulphur on the surface of ebonite, and that consequently all such insulators should be kept dark and dry, but a measure of exposure is almost unavoidable in practice. In any case solid insulators have to be handled during their construction, and their surfaces must therefore be cleaned subsequently. Thorough rinsing in distilled water and subsequent drying with a clean cloth or leather would appear to be a suitable treatment at this stage, and should prove equally satisfactory after a brief immersion in any electrolyte which has not had a permanent chemical effect on the ebonite.

Mr Hartshorn also mentions that the solid insulation may suffer a change on immersion. I believe I am right in stating, however, that such a stable material as quartz, which is widely used at present in condenser construction, would be quite unaffected by a brief immersion in a solution of salt and that all traces could be subsequently removed by efficient cleansing.

In answer to Prof. Hopwood: The complicated nature of the relationship between the concentration and the specific resistance of electrolytes at high frequency was known to me, and in consequence the actual value of ρ at each particular concentration, frequency and temperature was experimentally determined.

The President's remarks involve three separate points:

(1) The effect of the displacement current in the solution (as distinct from the conduction current), when the condenser is immersed, in altering the lines of

current flow in the plates and other metal parts. (2) The effect of the displacement current in the solution on the distribution of the displacement current in the solid dielectric and consequently upon the losses therein. (3) The effect of immersion upon the surface leakage on the solid dielectric. With regard to the first of these, the general theorem of the similarity of the lines of current flow between two equipotential surfaces immersed in electrolyte and the lines of the displacement current between similar surfaces in a dielectric appears to be well established (*vide* references on p. 127 of this paper). There are two factors determining exact equivalence in the two cases. These are firstly that the surfaces when immersed shall be equipotential, which is ensured to a high degree of approximation by the relative conductivities of copper and of the electrolyte used; and secondly that the boundaries of the electrolyte shall coincide with those of the electric field. Attention has been paid to these points in the paper. There seems to be no reason why the superposition of a relatively small displacement current in the electrolyte upon the conduction current should involve any alteration in the lines of current flow in it, in view of the fact that both follow similar paths. Consequently it should follow that the eddy-current losses in the metal parts of the condenser should be the same in both cases. So far as the second and third points outlined above are concerned, I have endeavoured in the present method to measure each of the two losses under conditions such that the other is negligibly small. Thus it will be seen from Table 4 that the shunt resistance equivalent to the insulation losses of this particular condenser was nearly 2 megohms at $f = 10^6 \sim$. Even if this resistance were reduced very materially by immersion, as would doubtless be the case, owing to alteration of the lines of the displacement current flow in the solid dielectric and to surface leakage, it would still have negligible effect upon the quantity under measurement, which is of the order of 0·1 ohm.

THE X-RAY STRUCTURE AND MAGNETIC PROPERTIES OF SINGLE CRYSTALS OF HEUSLER ALLOY

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ABSTRACT. The X-ray structure of single crystals of Heusler alloy has been examined by the single crystal rotation method. The alloy is found to crystallize as a body-centred cube with lattice constant 2·95 A.U. The aluminium atoms are distributed so as to lie on a face-centred cube with lattice constant = 5·9 A.U. Directional magnetic properties have been studied by a method previously described* and have been found to be identical with those of nickel which has a face-centred cubic structure. It is suggested that this may be interpreted as indicating that the manganese atoms—whose positions cannot be determined by X-ray analysis—also lie in a face-centred cubic lattice.

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§ 1. INTRODUCTION

SINCE the discovery of the Heusler† alloys in 1903, a great deal of work has been done upon them, particularly as regards the effect of heat treatment and ageing. Many of the researches have served mainly to emphasize the complicated nature of the problem, and very little direct evidence has been obtained concerning either the origin of the magnetizability of the alloys or the nature of the changes which take place on ageing‡. X-ray measurements by Young§ and von Harang‖ using the powder method have shown that the alloy crystallizes in the cubic system. Young found two types of crystal—a body-centred cube with lattice constant 3 Å.U. and a face-centred cube of 3·7 Å.U. L. von Harang found in addition to these two forms a cubic lattice with a cube side of about 8·70 Å.U.

It was thought probable that some valuable information might be obtained if single crystals of the alloys could be grown and examined both as regards crystal structure and magnetic properties. The simplifications introduced both into the magnetic and the X-ray measurements by the use of single crystals give some promise of useful results being obtained along these lines.

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    * *Science Abstracts*, no. 1399 (1928).
    † Heusler, *Phys. Gesel.* 5, 220 (1903).
    ‡ See, however, Kussmann and Scharnow, *Zeits. f. Phys.* 47, 770 (1928).
    § Young, *Phil. Mag.* 46, 291 (1923).
    ‖ von Harang, *Zeits. f. Krist.* 65, 261 (1927).

§ 2. PREPARATION OF CRYSTALS

Although the ternary alloys of copper, manganese and aluminium show measurable ferromagnetism over a considerable range of atomic proportions, the strongly magnetizable alloys belong to a fairly small region in which the approximate atomic proportions are two parts copper, one part manganese, and one part aluminium. Similar results are obtained if tin be substituted for aluminium.
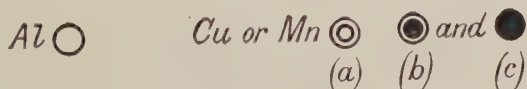
Owing to the lower melting points of the tin alloys these were tried in the earlier experiments, but so far they have yielded no single crystals. The aluminium alloys crystallize more readily, the process being as follows: Manganese is added to molten copper in a gas furnace in atomic proportions, one part manganese to two parts copper*. The binary alloy, which has a melting point of about 900° C., is then poured into moulds, weighed, and re-melted with the addition of aluminium in the proportion mentioned above. The alloy is well stirred and then poured into a mould, and after the grinding off of any scale the metal is re-melted in a vacuum furnace and cooled very slowly through the melting point. Some difficulty is introduced owing to the rather indefinite melting point and the strong tendency of the alloy to form scale; even when a rapid diffusion pump is used to evacuate the furnace, scale is readily formed (presumably by interaction of the metal with its own adsorbed gas), but two melts have been obtained which contained a number of crystals of useful dimensions. The presence of large crystals was tested by cutting sections, polishing and etching with ferric chloride. In this way single crystals up to roughly 150 mm.³ in volume were isolated. It is hoped that larger ones will be obtained later on, but it seems hardly likely that crystals of a ternary alloy will be comparable in size with those given by pure metals.

§ 3. X-RAY EXAMINATION

Seven such crystals—five from one melt and two from another—have so far been examined by the single crystal rotation method. The crystals can easily be identified as belonging to the cubic system and were, therefore, mounted with a tetragonal axis parallel to the rotation axis and the ordinary layer line photographs were obtained. The radiation used was the K radiation of either copper or molybdenum from a Shearer tube. In some ways the copper K radiation is inconveniently long, but it has several advantages over the molybdenum K radiation. Firstly, the Shearer tubes are more efficient with copper anticathodes than with molybdenum anticathodes. Secondly, in examining a crystal face of a highly absorbing substance like Heusler alloy, certain planes fail to give reflections owing to the reflected ray being in such a direction that it is unable to escape from the crystal. The shorter the wave-length the greater is the probability that this will occur. Fig. 1 indicates how the molybdenum radiation may fail to escape from a crystal, whereas with copper radiation, which has a bigger glancing angle, the reflected ray can emerge.

* In practice an excess of manganese is used to allow for oxidation. We are now, however, building an induction furnace and hope by vacuum melting to make more certain of our proportions.

All the crystals so far examined conform with the structure given in Fig. 2. The cube, which has an edge measuring 5·9 Å.U., contains sixteen atoms and can be divided up into eight smaller body-centred cubes having aluminium atoms at

Fig. 1. (a) Copper K radiation which can emerge after reflection. (b) Molybdenum K radiation which cannot emerge after reflection.

Fig. 2. Crystal structure of the Heusler alloys.

alternate cube corners. This smaller cube is similar to that obtained by Young and by von Harang, but whereas the latter speaks of the lattice as being homogeneous we have obtained definite evidence of the existence of planes of low scattering power which has enabled us to fix the position of the aluminium atoms. It is on

account of this that we have been compelled to choose a unit cell having twice the linear dimensions of that chosen by von Harang. For planes with three odd indices the identity period is four times that for the corresponding homogeneous lattice. If two indices be odd and one even, the identity period is unaffected by the heterogeneity. If two indices be even and one odd, the identity period is twice that of the corresponding homogeneous lattice. The high symmetry of the structure was verified by an X-ray examination of one of the crystals by rotation about each of its three tetragonal axes*.

It has not been possible to distinguish between the positions of the copper and the manganese atoms by means of X-rays. This would require a very careful investigation of the intensities of different order reflections, which would be made very difficult by the high absorption which takes place in the crystals. Some suggestions as to the possible positions of the manganese atoms will be offered in the discussion of the magnetic results.

In order to obtain single crystals it is not necessary to have exact atomic proportions. Several crystals were analysed, and all contained a slight excess of copper. Presumably the atoms are mutually replaceable in some degree at least.

§ 4. DIRECTIONAL MAGNETIC PROPERTIES

If a magnetic field be applied along one of the symmetry axes of the crystal the field and intensity coincide in direction, but if the field be applied in any other direction the intensity will not be along the field direction and may be resolved into a component (I_P) parallel to the field, and one (I_N) normal to the field. The methods of measuring these two components for varying direction of the applied field have been fully discussed in a previous paper†. Unfortunately, the ballistic method formerly used to study the parallel component in nickel was too insensitive for use with the Heusler crystals. This decrease in sensitivity was not wholly due to the Heusler crystals being less magnetizable than the nickel, but mainly to a reduction in size of the crystals, and the consequent increase in the proportion of "uncut" lines. It was necessary, therefore, to use the torsion method for the measurement of both the parallel and normal components of magnetization.

Now in view of a recent criticism‡ of the torsion method for the measurement of the parallel component it would at first sight appear that the use of this method is unsatisfactory. But it may be pointed out that in the case of nickel the results obtained by the two methods differed quantitatively rather than qualitatively. It will be seen below that the use of the torsion method in the present investigation is restricted to a comparison of the results for Heusler alloy with those of nickel. In that case the conclusions are not invalidated, although in its general application the method is unsatisfactory.

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* Since this paper was written, a letter by E. Persson has appeared in *Naturwissenschaften*, **31**, 613 (1928) on the X-ray examination of polycrystalline material. He reaches conclusions similar to ours.

† Sucksmith, Potter and Broadway, *Proc. Roy. Soc.* **117**, 471 (1928).

‡ *Ibid.* p. 482.

Three specimens prepared in disc form with the plane of the disc in one of the principal crystal planes have been examined. After the planes had been accurately located by X-ray measurements the discs were prepared by the method previously
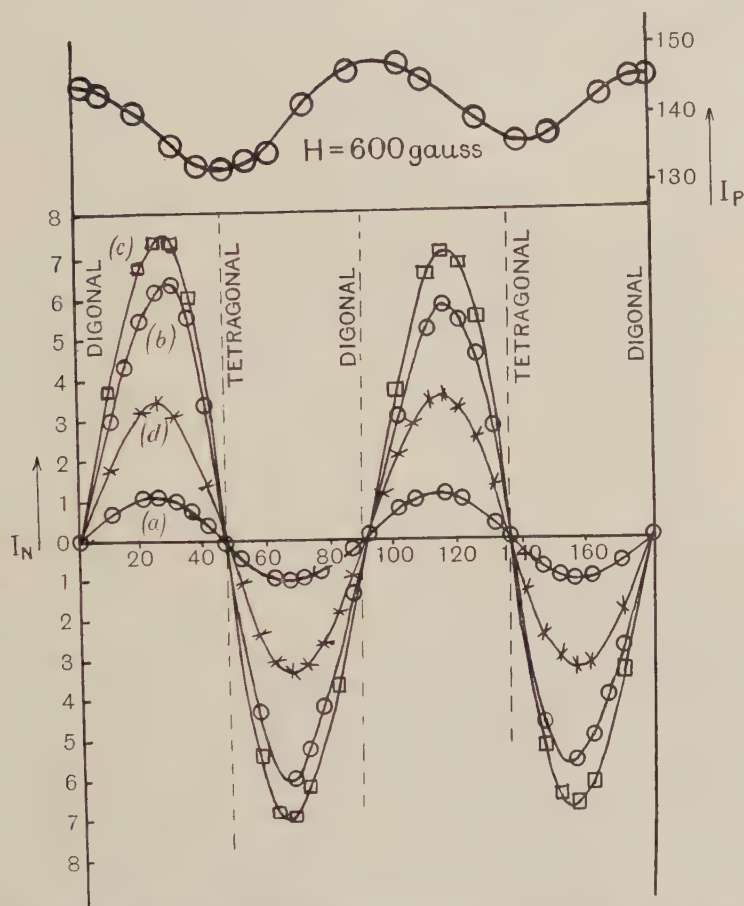

Fig. 3. Relation between magnetization and direction of field (abscissa) in {100} plane.
I_P, I_N, magnetization parallel and normal to field.
H = (a) 6500 gauss.   (b) 2180 gauss.   (c) 1110 gauss.   (d) 515 gauss.

described*. The diameters of the specimens used were the largest which could be produced. The results obtained in the three principal planes were as follows:

{100} Plane   Diameter of specimen = 0·280 cm.
                     Thickness   = 0·028 cm.
                     Weight   = 0·0112 gm.

The results in this plane are shown in Fig. 3. They are qualitatively identical with those obtained with nickel*, the maximum magnetizability being along the digonal

* Sucksmith, Potter and Broadway, *loc. cit.* p. 472.

axis, and the quasi-unstable position of the normal component along the tetragonal
axis.

{110} Plane   Diameter of specimen = 0·278 cm.
                     Thickness   = 0·025 cm.
                     Weight   = 0·0098 gm.

The results are shown in Fig. 4. Except for a somewhat more marked difference
in size between the large and small loops the curves are again very similar to those
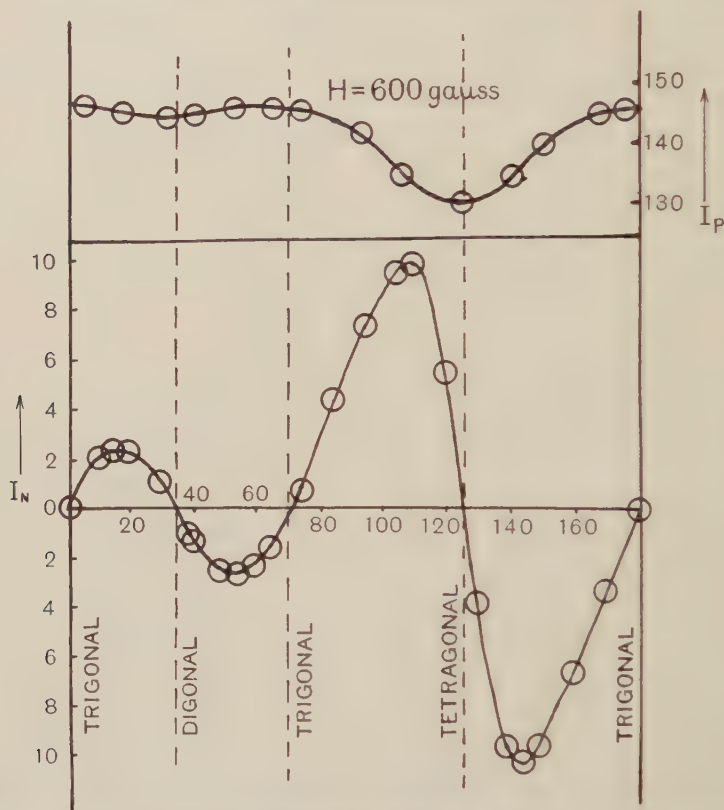
Fig. 4. Relation between magnetization and direction of field (abscissa) in {110} plane.
H=600 gauss for I_P, 1100 gauss for I_N.

obtained with nickel. The crystal magnetizes most easily along the trigonal axis
and least so along the tetragonal axis. In order to be certain of the reality of the
small dip in the parallel component curve, it was necessary to use an air core coil
in place of the electromagnet, the field of which was not reproducible with the
necessary accuracy.

{111} Plane   Diameter of specimen = 0·350 cm.
                     Thickness   = 0·038 cm.
                     Weight   = 0·0240 gm.

The curve for the normal component is shown in Fig. 5. Six complete loops appear in 180°. They are not equal in size, but since the maximum value of I_N is only about $1\frac{1}{2}$ per cent. of the total magnetization, slight distortion or inaccuracy in cutting the crystal would account for some irregularity in the curves. A 60° period for the normal component in the {111} plane of nickel has been obtained by Kaya*.

The fluctuations of the parallel component in the {111} plane were too small to be measured. They amounted to less than $\frac{1}{2}$ per cent. of the total intensity.
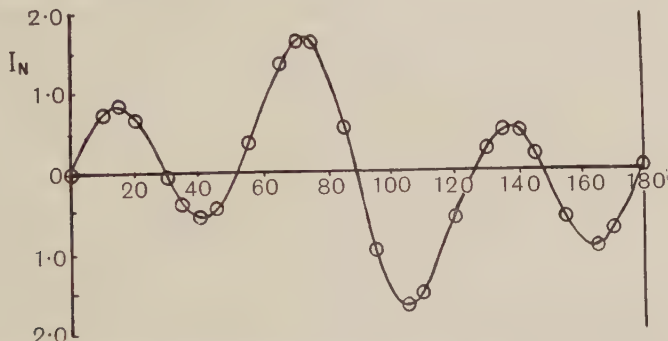
Fig. 5. Relation between magnetization and direction of field (abscissa) in {111} plane.
$H = 1600$ gauss.

§ 5. DISCUSSION OF RESULTS

The single crystals of Heusler alloy so far examined have all shown a body-centred cubic structure (cube side = 2·95 Å.U.), the aluminium atoms themselves forming a face-centred cube (cube side = 5·9 Å.U.).

The directional magnetic properties are almost identical with those of nickel, which belongs to the face-centred cube system, and bear no resemblance to the directional properties of iron† which like Heusler alloy has a body-centred cubic structure. This might be explained if the magnetization is influenced mainly by the distribution of the aluminium atoms. It appears more probable, however, that the important factor is the presence of the manganese, which itself has been obtained in a ferromagnetic condition‡ § and forms several ferromagnetic compounds. On this assumption the directional magnetic properties would probably depend on the spatial distribution of the manganese, present possibly in some modified form which gives it ferromagnetic properties. It seems probable, therefore, that the manganese like the aluminium is arranged in a face-centred cubic structure. This can occur in more than one way while the general structure shown in Fig. 2 is still preserved. Firstly, the circles indicated by (a) and (b), Fig. 2, may be taken to represent copper atoms, and the circles indicated by (c) manganese atoms. The

* *Sci. Rep. Tôhoku,* **17**, 639 (1928).
† Honda and Kaya, *Sci. Rep. Tôhoku,* **15**, 721 (1926).
‡ Seckelson, *Wied. Ann.* **67**, 37 (1889).
§ Wiess and Onnes, *Compt. Rend.* **150**, 689 (1910).

copper atoms would then form a simple cubic system. This arrangement satisfies density considerations, chemical proportions, and X-ray data. If this structure be correct the elementary cubes (of which there are eight in the figure) would have copper atoms at the centres and aluminium and manganese atoms at alternate cube corners. There is, however, at least one other method in which the manganese atoms may be placed in a face-centred cubic lattice without disturbing the general atomic arrangement. The atoms indicated by (*b*) and (*c*) in Fig. 2 may all be copper and the atoms marked (*a*) may be manganese. We should then have four inter-locking face-centred cubes, two of copper and one each of manganese and aluminium. It is unfortunate that X-ray intensity measurements cannot so far be made accurately enough to distinguish between these two structures. The work is being continued, and amongst other things it is hoped that it will be possible to study the effects of heat treatment and variation of atomic proportions.

§ 6. ACKNOWLEDGMENTS

A FUSED QUARTZ PENDULUM ROD FOR CLOCKS

By C. V. BOYS, F.R.S.

Received November 27, 1928. Read and discussed January 25, 1929

ABSTRACT. The author discusses the possible causes of the progressively increasing losing-rate found in the going of the Shortt clock. He shows a design for the free pendulum suitable for use where the rod is made of fused quartz. He would prefer tempered carbon steel for the supporting springs.

I T may be within the recollection of some members of the Physical Society that in the year 1917 Mr C. O. Bartrum read a paper* before the Society in which he described a free pendulum with slave clock which he had devised and constructed. In this he made an important advance in obtaining a pendulum maintained in motion by uniform gravity impulses delivered once a minute, and having no work whatever to do beyond that inherently necessary in overcoming the imperfection of elasticity in the supporting spring and the viscosity of residual air in an enclosed case. A most important feature of his design was the release of the impulse weight after its operation on the free pendulum to direct the slave clock to hurry or to lag to an infinitesimal extent so as to keep it strictly in phase. The slave clock of course released the impulse device. In other respects the design was unnecessarily complex, but it worked.

In more recent years all previous clock performance has been completely eclipsed by the clocks designed by Mr Shortt and made by the Synchronome Company, and now invading all the observatories in the world. The design of this clock is notable for its extreme simplicity and excellence. Its going is so perfect that its day by day indications are more uniform than day by day intervals determined by star transits. The Astronomer-Royal for Scotland has made a very extensive study of the behaviour of this clock, and two are in operation in our own Royal Observatory. From these observations it would appear that these clocks show a very small but progressively increasing losing-rate, and there is some doubt as to its cause. On the whole the invar pendulum rods are most under suspicion, under the belief that they have the property of growing in length to a very small but real extent in service. Suspicion is also cast upon the elinvar suspension spring.

I have some doubt myself whether the nickel steel alloys are either of them the cause of the trouble. Mr Hope Jones informs me that the bobs of these pendulums have been made of type metal†, undoubtedly good in an ordinary way. Pure lead

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* *Proc. Phys. Soc.* **29**, 120 (1917).

† Type metal is probably the best material for the bob, but non-magnetic steel might be worth consideration for this purpose. C. V. B., January 11, 1929.

I should of course condemn, and I have advised a friend to replace a lead bob in a precision clock by one made of gun metal but not of any lead alloy, for the sake of greater stability. As the density of the bob is not of much importance, the pendulum being enclosed in a nearly exhausted case, I should prefer to cut the bob from a bar of rolled mild steel annealed after roughing to form and then finished. The very heavy alloy of tungsten and iron—which is easily made by the thermit process, has a density in the neighbourhood of sixteen, and works beautifully in the lathe—would be worth considering if the pendulum swung in air at full pressure, but the cost and uncertainty attaching to any unproved alloy rule it out for the present purpose. The type metal may or may not be right, but it is a simple thing to try steel. Unfortunately experiments of this kind require some years for completion.

Taking the three elements—rod, spring, and bob—mentioned above, my own view is that in respect of the probability of defect they must be arranged in the following order:

1. *Spring.* Under considerable tension and constantly being bent, but very short.

2. *Rod.* Under far less tension relative to its cross-section and never being bent, but long.

3. *Bob.* Under infinitesimal stress by comparison and short, but of poor material.

At a meeting of the Astronomical Society last winter* I expressed the view that I should have more faith in a carbon steel rod than in one of invar, but that fused quartz would be the best possible material. It is now forty years since I showed that fused quartz is the one material which possesses all the physical virtues and is devoid of vice, and I believe that subsequent experience has confirmed this view. I find now that in more than one quarter the introduction of a fused quartz pendulum rod is being considered and that one design that has been contemplated is in my opinion hopeless. It seems to me therefore that it is desirable to publish the scheme which I had in mind at the beginning of 1928. The design must be adapted to the properties of the material, with which I am peculiarly well conversant, and need in no way follow that which is suitable for a metal rod.

In Figs. 1–4 I have shown the attachment at the top end, and in Figs. 5 and 6 the attachment to the bob. Fig. 1 is a vertical section through the supporting springs *S, S*. Fig. 2 is a plan with the amplitude mirror *M* removed. Fig. 3 is a side elevation. Fig. 4 is a plan of the clamping piece *C* alone. The rod *R* is formed with a boss at each end, and these are ground in a proper grinding lathe so that the cylindrical neck, head and cone are true to a single axis. The clamping piece *C* is saw cut as shown in Fig. 4 to enable it to be put on to the rod and to be pinched on to it. The two inclined saw cuts should be made before the piece is finally cut to form and the free piece should be sweated in place. Then after it is accurately finished in the lathe the piece may be removed by gentle heat and the films of

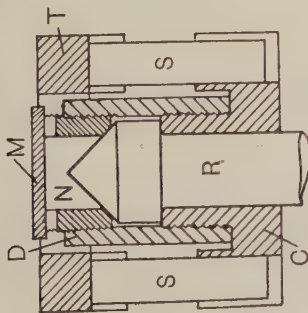* *The Observatory*, **51**, 83 (1928).
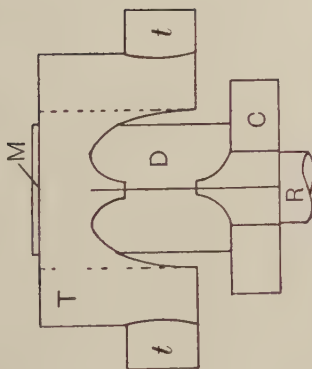
Fig. 5. Elevation, mainly in section, of the bob.

Fig. 6. Horizontal section at dotted line.

Fig. 3. Side elevation of top.

Fig. 4. Plan of clamping piece C.

Fig. 1. Elevation, mainly in section, of top of pendulum.

Fig. 2. Plan of top, with mirror M removed.

solder smoothed down. In this way gaping saw cuts are avoided*. The two parts of the clamping piece are held together by a tube D gradually screwed down while it is being ground with fine carborundum up and down the rod and on to its seat. When ultimately the tube D is screwed down securely the sloping faces of the screw thread will cause the clamping piece to grip the rod, and the nipple N screwed in firmly will make it certain that the shoulder under the head is firmly upon its seat. The springs S, S which carry the pendulum are a tight fit in the saw cuts in the clamping piece and in the trunnion gimbal T which, carried by the actual trunnions t, t, ensure that the two supporting springs are equally loaded. The supporting springs are sweated in place with solder rich in tin exactly as I described in my article on the grandfather clock†, and the observations I there made on this mode of attachment are, I hope, sufficient to overcome the prejudice against soft solder properly used, which is simply due to ignorance. I should prefer tempered carbon steel to any alloy for these springs, and non-magnetic steel for the whole of the parts at the top end. It will be seen then that the trunnion axis, the plane of attachment to the rod and the centre of the springs are all at one level and that differential expansion is eliminated.

Figs. 5 and 6 show the attachment of the bob. Here the clamping piece is made in two halves sawn after roughing, sweated, finished to form and separated again and ground to the rod, all as already described for the upper clamping piece. Here again the plane of resting is exactly defined and the rod is fixed centrally by the action of the nipple N bearing on the cone of the rod and elbowing the halves of the clamping piece against the inner wall of the bob. A pinching nut p both causes the clamping piece to grip the rod at the level of the nut and ensures firm contact between the supporting shoulder of the clamping piece and the bob. The rod and bob are thus relatively defined in position. I have shown this shoulder a certain distance x above the quartz shoulder and the quartz shoulder a certain distance y below the centre of oscillation of the bob. These two distances x and y can only be determined when the material and size of the bob are decided upon. It will be seen that the distance x is available for the purpose of any compensation which may be necessary, and the material of the clamping piece will be determined accordingly. The screwed hole at the lower end of the bob is available to take a timing plug or to hold any device which it may be desired to attach at the lower end of the pendulum.

While of course there is no absolute proof that a fused quartz pendulum rod will not gradually lengthen in service, the extremely perfect behaviour of this material and its very simple constitution and high melting-point all lead to the conviction, amounting practically to a certainty, that it will not.

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* The clamping pieces should be of non-magnetic steel. The saw cuts for the springs should be not less than 1 16th in. thick, so that they may be milled with accuracy with a proper milling tool in a machine. The spring must be made up to the required thickness at each end by folding a strip of tinned brass (or even iron) over it so as to ensure a good fit. This arrangement will also facilitate sweating, which must be done in a jig. C. V. B., January 11, 1929.

† *Journ. Sci. Instr.* 3, 143 (1925).

DISCUSSION

Mr A. CAMPBELL (communicated): I think that Prof. Nagaoka told me in 1924 that he had constructed a pendulum of fused quartz. In his paper on pendula* he advocates the use of tungsten as a specially suitable material.

Dr D. OWEN: I recall the meeting of this Society at which one of our Fellows, Mr Bartrum, read his paper (to which Prof. Boys so eulogistically referred in the present paper) on the subject of a precision clock consisting of a free pendulum and a slave pendulum. That paper was perhaps the first scientific publication embodying that fruitful idea. Prof. Boys, then our President, was in the Chair on that occasion, and took part in the discussion that followed, as did also another great experimental physicist, the late Mr Duddell. It appears from the present paper that Prof. Boys, in order to secure mechanically good material, has no compunctions about employing mild carbon steel for the bob and for the mountings at the top and bottom of the quartz rod of his free pendulum, although the coefficient of expansion of this steel is large and it is highly magnetic. In regard to the expansion effect, presumably the differential action allows of virtually perfect compensation. But it appears as if the magnetic effect might introduce a control couple amounting to several parts in a million of that due to gravity, so that the effect of secular change in the vertical component of the earth's magnetic field might in time be not inappreciable. Doubtless this field might be compensated, should this be really necessary.

Dr J. H. VINCENT: I think that experiment alone will be competent to decide whether fused quartz extends under long continued tension. The remarkable physical properties of this material seem at variance with the view which regards it as merely an exceedingly viscous solid. It may turn out that even fused quartz will flow under tension to an extent which the very sensitive test of the pendulum will render evident. But even so it might make a better pendulum than invar: this, being an electrical conductor, will introduce a quasi-viscous effect when swinging in a magnetic field; and it is also magnetostrictive so that its length is a function of the magnetic field. In a free pendulum I suppose the greater the freedom the better. If this is so, it becomes a matter of moment to determine the actual paths of escape of the small periodical doses of energy. It looks as if not only will the free pendulum have to be designed so as to lessen these losses by such means as having double pendula swinging in opposition on a common support, but small effects hitherto tolerated must be eliminated. The pendulum though not conducting should be coated with a very thin layer of a conductor which should be of the same material as the inside of the case, thus keeping the case and its contents at the same electrical potential. The air in the case should not only be at uniform temperature and pressure, but of constant humidity. Finally the case and its contents should be made a region of zero magnetic field.

* *Phys. Math. Soc. Japan,* **1,** 347 (1919).

Mr C. O. Bartrum: I am unable to express any opinions on matters where the properties of materials are involved. With reference to the steadily increasing losing rate of the Shortt clocks at Greenwich, I may say that Dr J. Jackson, Chief Assistant, who is devoting special attention to these clocks, is coming to the opinion that the change is too great to be attributed only to the lengthening of the invar rods. From the text of the paper it seems that Prof. Boys also has doubt on this point. Dr Jackson is looking for some creeping of the bob and for a gradual movement upwards of the instantaneous axis of bend of the suspension spring.

To those who read astronomers' reports of the rating of clocks it may be pointed out that astronomers make strange use at times of algebraic signs. As when a star decreases in brightness its magnitude increases, so when a clock goes more slowly its rate is said to increase.

Dr Owen's comment on the proposed use of steel with its high coefficient of expansion with temperature in the head of the quartz pendulum is answered I think by the design, in which the steel is so disposed that its expansion cancels out.

Dr Vincent referred to various possible causes of dissipation of energy of the pendulum. Mr Shortt made experiments to determine the effect of reduced air pressure, and he found that the curve of dissipation plotted against density converged towards a point that was not zero with complete evacuation. This residual he attributed to molecular dissipation in the suspension spring.

Mr R. Appleyard: Apparatus constructed by Prof. Vernon Boys is always characterized by precision beyond the range of prudent criticism. In this pendulum, as in all his work, the precision is not the result merely of extended arithmetic applied to correction factors—it is the reward for extreme care in the choice of materials, in the design of parts, in the selection and use of tools and processes, and in the fitting and manipulation of the whole product of his transcending genius. A few weeks ago I came by chance upon some unpublished notes that passed between Francis Ronalds, of Kew Observatory, and Airy, the Astronomer-Royal, that bear upon the history of the growth of such correcting devices. On January 26, 1847, Airy wrote from Greenwich: "I shall be curious to see your barometer-self-corrected-for-temperature arrangement. There is commonly a fear of failure in self-acting corrections of this character, but I do not know that it is well founded." And again, on February 12, 1847, after mentioning the complication of the machinery of Ronalds' apparatus and the fear that it might go wrong, he added: "The gridiron pendulum is not often to be trusted although it generally acts tolerably well." Airy also warned Ronalds that the law of expansion of metals is not accurately known and is not truly linear. He suggested that a record of the temperature should be kept. The truth is, he realized that even metals and their joints are only human.

Prof. F. L. Hopwood: Under the action of intense ultra-violet light fused quartz slowly becomes crystalline with consequent changes in dimensions. Whether the amount of ultra-violet light falling on a pendulum of fused quartz would be sufficient to produce a sufficient change in length to affect its rate appreciably, I do

not know. I would like to ask Prof. Boys where one could obtain the steel of specific gravity 16 which he mentions in his paper.

Mr F. A. GOULD: It occurs to me that the use of fused quartz as the material of the pendulum rod of a high-precision clock may be limited by the presence of electrostatic charges on the rod. Crystal quartz is easily electrified and is very difficult to discharge completely even when X-rays are used for this purpose. I do not know whether fused quartz behaves similarly in this respect, but I should be interested to know whether Prof. Boys has encountered any difficulties of this nature during his long experience with the material. It ought not to be impossible to overcome the difficulty in the event of its interfering with the accuracy of the fused quartz pendulum rod.

Prof. D. ROBERTSON: Prof. Boys' paper has been well timed, for the information recently published anent the going of the Shortt clocks at Greenwich and Edinburgh has probably set others besides myself making enquiries as to why quartz was superseded by invar as the standard material for pendulum rods. I would like to suggest that the plane at which the rod is supported should be higher than the centre of the spring, so that part of the temperature compensation may be given at the top. In a forthcoming article (probably in April) in a series now running in the *Horological Journal*, I have analysed the temperature errors of a particular pendulum and estimated their location as follows:

At the top	— 190 ms/d per 1° C.	(A positive error means that
Distributed	— 46 ,,	the clock goes fast with
At the bottom	+ 46 ,,	rise of temperature)

These figures apply to running in a sealed case; if the case be open to variations of barometric pressure, the bottom error becomes + 87.

To reduce the effects of stratification to a minimum, I advocate that the compensator at the bottom should correct only for the bottom error and half the distributed one; the remainder should be dealt with by a second compensator at the top. This is quite easily provided in the design given by Prof. Boys when modified as suggested above.

AUTHOR'S reply: I have to thank Dr Owen for kindly reading my paper in my enforced absence.

Of course it is imperative that all massive material in the pendulum should be non-magnetic, but the tempered carbon steel supporting springs seem to me to be essential.

The point raised by Dr Vincent as to the energy lost by eddy-currents due to the rotation of the bob in the earth's magnetic field is a real one. Whether or not the loss is comparable to the energy which has to be provided, my imagination cannot answer. The loss might be calculated, or as it is proportional to the square of the strength of the field a test could be made very easily by fixing a pair of long thin magnets in the line of the dip, one on each side of the clock so that the field in which the bob moves is reduced to a small fraction of the earth's field. Then in

the course of a few days an increase in amplitude would demonstrate the existence of this loss and its amount. I do not like the double pendulum suggestion. The fused silica rod has one physical virtue which in the present case is a nuisance. It is a perfect insulator* of electricity or nearly so and to prevent electrification due to any unknown cause it might be necessary to silver it.

In answer to Prof. Hopwood: The piece of tungsten-iron alloy rich in tungsten was given me by General Holden, F.R.S., and I turned it and "played with it." As I said, it is easily made by the thermit method, but I must warn anyone playing with thermit mixtures that there are dangers, as I have seen.

I should be only too glad if Prof. Robertson would go into the detail design and by all means counteract stratification by distributing compensation. There should as a fact be no stratification because no one using a Shortt clock is treating it fairly if he does not have it in a constant-temperature room in which strong air circulation is maintained all the time.

May I in conclusion say that the centre of gravity of this paper is the mode of attachment to the rod of the solid metal parts without shake, without strain and about defined planes; and a minor point is the defiance of the belief prevalent in the horological world that a double suspension spring is undesirable.

* *Proc. Phys. Soc.* **10**, 128 (1889).

A SURVEY OF HEAT CONDUCTION PROBLEMS

By EZER GRIFFITHS, D.Sc., F.R.S.

Lecture delivered on December 14, 1928

ABSTRACT. Several forms of thermal conductivity apparatus are described which have been devised for the study of diverse materials ranging from a sheet of mica to a wall section weighing half a ton. The materials are broadly classified into four groups:

(1) Materials of very low thermal conductivity, such as cold storage insulators. (2) Materials supplied in the form of thin sheets and those employed in building construction. (3) Refractories and materials employed in furnace construction. (4) Pure metals and alloys.

Samples of baked slab cork are shown with a conductivity as low as 0·00007 c.g.s. unit*. The insulating value of such materials is due to the fact that they subdivide the air space into a large number of minute air cells. The theory of heat conduction through granular material is an interesting problem. An attempt to work it out mathematically has been made by M. Smoluchowski†. The wide divergence between the results obtained by experiment and those calculated from theoretical considerations indicate that the mechanism of the heat transfer from solid to gas is one which requires further study.

In the testing of materials of low thermal conductivity by the hot plate method attention has to be given to heat leakage from edges and corners. The simplest procedure is to eliminate the effect by the use of a guard plate, but it is also possible to calculate an approximate correction for the "edge effect" when using the simple hot plate method. A mathematical problem awaiting solution with numerous practical applications is the adaptation of the method of conformal representation to three-dimensional problems. Sixty years have now elapsed since the publication of the important papers by Schwarz and Christoffel, and in this interval numerous papers have appeared demonstrating what a powerful tool conformal representation is for the solution of two-dimensional problems. Substances in the form of thin discs can be tested by the divided bar method, the correction for the thermal resistivity of the two mercury films being obtained by independent experiments using a thin disc of iron instead of the specimen. This apparatus is not well adapted for the study of poor heat-conductors, owing to the fact that the temperature drop across the specimen is then large and the gradient along the two bars small. The electrically heated plate, sandwiched between two discs of the material with water-cooled plates on either side, is an apparatus which can be adapted to various substances obtainable in sheet form. A form of this apparatus devised for the study of the effect of pressure on the flow of heat through a pile of paper-covered iron stampings is illustrated. In this test the thermal resistance across the laminations is measured. For the determination of the heat flow along the laminated material a bar about 15 in. long of square section, 2 in. by 2 in., is built up. One end is electrically heated, while the other dips into a pot of mercury carrying a cooling coil. From the temperature gradient and the heat flow the conductivity can be calculated.

The apparatus for the study of refractories consists of a furnace made up of "silit" rod heaters which can raise one face of the slab under test to any temperature up to 1000° C. On the top of the slab is a water-flow calorimeter fitted with a guard ring.

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* The conductivity of air is about 0·00006.
† *Bull. Intern. Acad. Sciences*, Cracovie, 5 A, 129 (1910) and 8 A, 548 (1911).

The calorimeter and guard ring are made of thick slab brass with grooves milled in the back surfaces for water circulation. To reduce heat transfer between calorimeter and guard ring the edges of the plates are chamfered off so that the edges on the brick are near together, whilst the rear parts are well separated. For the study of metals and alloys the guard tube method for the elimination of heat loss sideways is to be recommended in preference to Forbes' procedure. The space between guard tube and specimen is packed with a powder of low thermal conductivity. Both specimen and guard tube can be attached to a cylindrical block containing a heating coil. The heat flow is determined with a cooling coil fixed to the other end of the specimen. Water flows round the end of the specimen and the guard ring so that the same gradient is maintained automatically in specimen and guard tube. In some experiments the guard tube is dispensed with and the heat leakage through the lagging material calculated approximately.

In the study of two groups of alloys—aluminium alloys and bronzes—it has been found that the two groups give values of the Lorentz coefficient approximating to the values for the pure metals which form the principal constituents of the alloys—namely, 5·5 for the aluminium alloys and 5·9 for the copper alloys.

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§ 1. INTRODUCTION

IN this lecture I do not propose to deal with the purely theoretical problems in heat conduction which are to be found in abundance in Carlslaw's *Heat Conduction*, Byerly's *Fourier's Series and Spherical Harmonics*, and Ingersoll and Zobell's *Mathematical Theory of Heat Conduction*, but rather with the practical difficulties of the worker who has to determine the thermal conductivity of a material in whatsoever form it is supplied to him. When you have to tackle materials ranging from a sheet of mica to an earthenware duct you never lack variety or troubles!

I propose to give a short survey of the various forms of apparatus which we have employed for the measurement of thermal conductivity, and of their peculiar defects and advantages.

No particular novelty is claimed for the methods, but possibly the experience gained in the course of the study of the most diverse classes of materials may be helpful to others confronted with similar problems. For the purposes of discussion it is convenient to classify materials into the following four groups:

(*a*) Materials of very low thermal conductivity, of the order of 0·0001 c.g.s., such as are used for "cold storage" insulation.

(*b*) Non-metallic substances obtainable in the form of thin sheets of medium thermal conductivity of the order of 0·002 c.g.s. and less.

(*c*) Refractories and materials used for heat insulation at high temperatures. (Thermal conductivity from 0·0002 to 0·005 c.g.s.)

(*d*) Metals and alloys. (Thermal conductivity from 0·6 to 0·1 c.g.s.)

I shall deal with each of these groups in turn and indicate the special points needing attention in their study.

§ 2. MATERIALS FOR COLD STORAGE INSULATION

The characteristic of these materials is their low thermal conductivity, a coefficient of thermal conductivity of 0·0001 gm. calories per cm². per sec. for 1 cm. thickness and 1° C. difference between the faces being usual for a high grade material. This would be for a cold face temperature of 0° C. and hot face of 15° C. The conductivity increases with temperature. Baked slab cork is obtainable with a thermal conductivity as low as 0·00007 and, when we remember that still air has a conductivity of 0·00006, it will be realized how efficient these materials are. Their insulating value is due, not so much to the fact that the materials themselves are poor heat-conductors, as to the fact that they subdivide the air space into a large number of minute air cells.

Fig. 1 is a microsection of a material composed of gas cells enclosed by membranes of hard rubber.
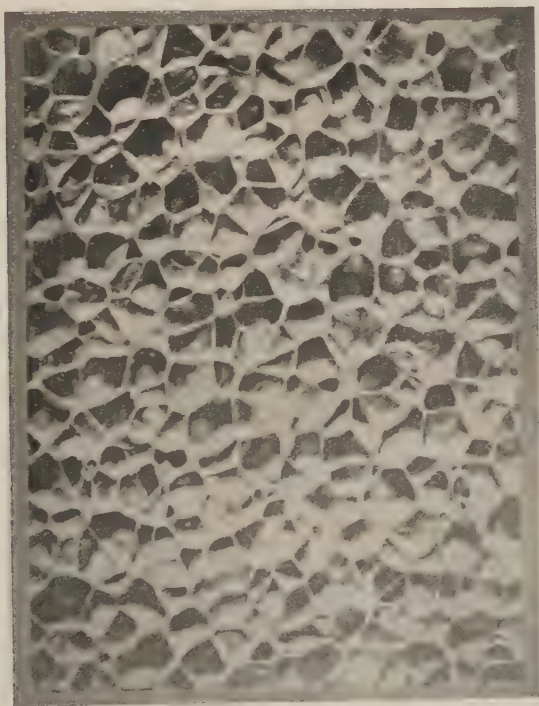
Fig. 1. Microsection of rubber insulating material.

The ideal material for cold storage insulation would be an assemblage of minute gas cells, totally enclosed in a light framework impervious to moisture. The ideal material has not yet been found, but high grade cork granulated, pre-heated, steamed and then run into moulds, and heated under pressure to about 250° C. gives a slab which is a remarkably good insulator, and in practice a material impervious to water is subsequently applied to the surfaces.

Thermal conductivity data for cold storage insulation

It may be of interest to quote here a few values selected from the variety of materials which have been studied. In each case the temperature range is from about 0° to 16° C., although data have also been obtained for various mean temperatures up to 100° C. when maintaining the same difference between hot and cold faces throughout the range.

Table 1

Material	Conductivity c.g.s., according to density
Baked slab cork...	0·000072 to 0·0014
Granulated cork	0·00008 to 0·00011
Cork shaving such as is used for cigarette tips	0·000073
Diatomaceous earth	0·000193
Slag wool, density 15 lb. per cu. ft.	0·000102
Cellular expanded rubber sheet	0·00009
Ditto. Clippings packed to 4 lb. per cu. ft. ...	0·000106
Cellular expanded ebonite, 5·4 lb. per cu. ft.	0·000090
Rubber sponge, 14 lb. per cu. ft.	0·00013
Peat dried and compressed	0·00009 to 0·00018
Pumice in granulated form	0·00022

The above list could be extended to many times its length, but it will suffice to illustrate the order of magnitude of the conductivities.

Theory of heat conduction through granular material

The theory of heat conduction through powders and granulated substances has been worked out mathematically by M. Smoluchowski*, who assumes:

(1) The powder to be made up of perfectly conducting spheres arranged in cubical order.

(2) That there is a discontinuity of temperature at the interface between solid and gas.

This last assumption is rendered necessary by the fact that there would otherwise be infinite flow of heat near the point of contact of the spheres. Whilst his mathematical work is of fundamental importance, the experiments which he made to verify the formulae were somewhat crude. He used two concentric cylinders, the inner being the bulb of a mercury thermometer whose rate of cooling was observed while the outer cylinder was in ice. The gas pressure was varied from 22 to 760 mm. However, he showed that the conduction through the gas in the interstices of the powder was given by the expression

$$K = \tfrac{1}{2}\pi k_0 \log_e (1 + a/\delta),$$

where k_0 is the conductivity of the gas,

  a is the radius of the sphere,

  δ is the mean free path, λ, of the gas molecule multiplied by a factor determined by experiment.

He found δ to be 1·2λ for air and carbon dioxide, and 2λ for hydrogen.

* *Bull. Intern. Acad. Sciences*, Cracovie, **5 A**, 129 (1910) and **8 A**, 548 (1911).

In the second paper Smoluchowski worked out the theory of conduction through a powder in a vacuum. The introduction of spheres of radii a between the walls d cm. apart will reduce radiation from R to $R.2a/d$. If d be large compared with a the radiation transfer will be small. He prepared zinc powder by distillation and found it composed of spheres of fairly uniform size 0·028 mm. He then computed the area of contact by Hertz's theory and, assuming perfect thermal contact, found the calculated conductivity to be about 1000 times greater than that found by experiment. He concluded that good thermal contact did not exist between the grains. The mechanism of the heat transfer from solid to gas is one which requires further investigation; it is, for example, conceivable that in the above experiments the zinc spheres were coated with a film of oxide.

Aberdeen and Laby* studied experimentally the effect of gas pressure on the conductivity of a very light powder, silox, in air, carbon dioxide and hydrogen at pressures varying from 1 to 760 mm. They found the empirical expression

$$k = \tfrac{1}{2} k_0 \log_{10} (p/n)$$

to represent their results, where

k_0 is the thermal conductivity of the gas,

p its pressure, and

n is a constant for the gas.

This expression approximates to Smoluchowski's. It might be noted that these investigators obtained a conductivity value as low as $0·7 \times 10^{-5}$ c.g.s. for silox in the best vacuum attainable in their apparatus. The use of such a powder in the evacuated space between the walls of metal Dewar vessels would render high polishing of the surfaces unnecessary.

Apparatus for determination of the conductivity of cold storage material

In tests of coarsely granular material it is necessary to use a test sample whose thickness is large compared with that of the granules. The use of a large sample has the further advantage of giving an average value. The standard form of apparatus employed consists of a hot plate sandwiched between two cold plates. With materials of low thermal conductivity it is essential either to employ a guard ring round the hot plate so as to ensure that the flow takes place along the prescribed paths, or else to make a correction for side loss. It might be remarked here that when a hot plate is inserted between two thick walls of insulating material a calculation based on the assumption that the heat will flow out normally to the hot plate may give results in error by as much as 45 per cent. owing to the leakage of heat from the edges†.

One form of apparatus employed has a 3 ft. by 3 ft. hot plate made of aluminium sheets clamped on to a flat heating coil. The guard ring is 1 ft. wide and separated by a narrow air gap from the hot plate. The two cold plates consist of steel plates

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* *Proc. Roy. Soc.* **113 A,** 459 (1926).

† This is true of a plate 3 ft. by 3 ft. surrounded on all sides by an 8 in. uniform thickness of insulating material.

5 ft. by 5 ft. with a grid of pipe screwed on to the back of each. The entire apparatus is slung so that the plates can be moved from the vertical to the horizontal plane. Testing a layer of material both vertically and horizontally will show whether an appreciable amount of convection is taking place in the interspaces. Thermocouples of twin laid copper constantan are used for the measurement of the temperature at various points on the hot and cold faces.

Most of the conductivity measurements on cold storage materials can also be carried out with the apparatus shown in Figs. 2 and 3. The test samples in the form of slabs are 12 in. by 12 in. by about $1\frac{1}{2}$ in. thick, while powders or granules are enclosed in a frame. The resistor elements of both the hot plate and guard ring are made of nichrome strip threaded through suitably spaced slots in a sheet
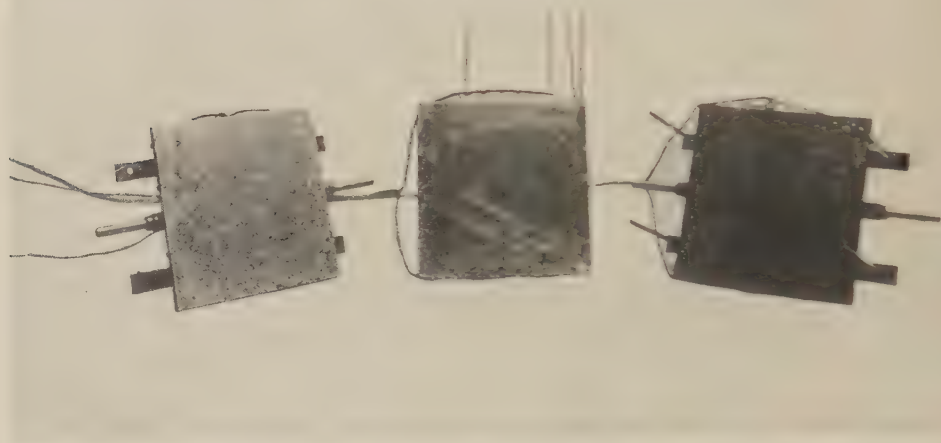
Fig. 2. Electrically heated hot plate with guard ring and two cold plates.

of micanite. Both hot plate element and guard ring are carried on the same sheet of micanite, which is cut away in the region between the hot plate and guard, leaving only narrow connecting tongues so as to ensure maximum thermal resistance. The aluminium plates are clamped on to this micanite sheet, electrical insulation from the winding being effected by exceedingly thin sheets of micanite. A series of copper constantan thermojunctions are attached to the inner surfaces of the metal plates by lead plugs and the wires are carried out in grooves milled in the plates. The cold faces are made up of two surface plates maintained at a constant temperature by water circulation. Surface plates can readily be converted into hollow boxes. Holes are drilled into the plates parallel to the surface and thermocouples are inserted in these.

The arrangement for circulating water or brine at constant temperature through the cold plates needs no explanation. In thermal work the guard ring manipulation is not such a simple matter as in the corresponding electrical problem, for the temperature has to be built up by the dissipation of energy and not merely by

coupling to a fixed potential. The use of a guard ring necessitates some care in adjustment of the temperature of the ring to equality with that of the hot plate. Usually it is possible to make the adjustments so that in one experiment the temperature of the guard ring is slightly higher than the hot plate and in the next that it is slightly lower. If a guard ring be dispensed with it is absolutely essential to allow for the flow sideways when testing thick layers of materials of medium conductivity. The two-dimensional flow is amenable to rigorous calculation by the method of conformal representation, but in practice the flow is in three dimensions, and the three-dimensional flow at a corner cannot be calculated. Another objection to dispensing with the guard ring and using a more complicated formula is the fact that, if uniformity of temperature over the hot plate is to be secured, thick metal plates are necessary; otherwise the increased heat flow at the edges of the hot plate introduces gradients across it.
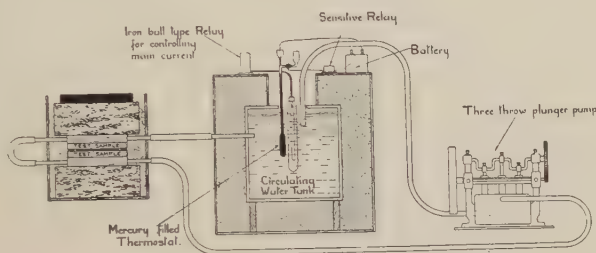
Fig. 3. Arrangement of apparatus for maintaining steady temperatures at the cold faces.

Practical considerations, however, sometimes necessitate the use of a simpler form of apparatus, as, for example, in testing wall sections whose conductivity may be twenty-fold that of cork. In this case the apparatus consists of a hot plate laid on a slab of cork with the test specimen laid on the hot plate. No guard ring is employed with this apparatus, but the heat flow through the cork packing around the edges is allowed for approximately. The hot plate rests on a slab of baked cork about 7 in. in thickness. Disc thermocouples of copper constantan are stuck on to the hot and cold faces of the wall section. The heat flow through the slab of cork covering the back is readily computed, but the calculation of the heat flow sideways is a difficult problem, as assumptions have to be made which can only be roughly correct. For instance, the boundary has to be assumed an isothermal. By making the dimensions of the hot plate large in comparison with the thickness of the material, the edge correction can be made very small—of the order of a few per cent.—in comparison with the flow through the test sample.

"Box" methods of conductivity measurements

The use of a box containing a heating coil is sometimes advocated for conductivity work, the walls of the box being composed of the material under test. The serious objection to that form of apparatus is that it is not possible to obtain an exact mathematical expression for the heat flow through a geometrical system

of this form. Even for approximate calculations the assumption has to be made that the surfaces are isothermal—a condition which is rarely realized in practice. Probably the simplest method of dealing with a complicated shape is not to attempt a mathematical solution, but to construct an electrical condenser of the same form and determine its capacity with air as dielectric.

Two-dimensional heat flow problems

It is disappointing that no one has hitherto adapted the method of conformal representation to three-dimensional problems. For two-dimensional heat flow problems the theorem of Schwarz and Christoffel* is a powerful tool for their solution†, but in thermal conductivity measurements we usually have to deal with a three-dimensional problem. Schwarz published his important paper sixty years
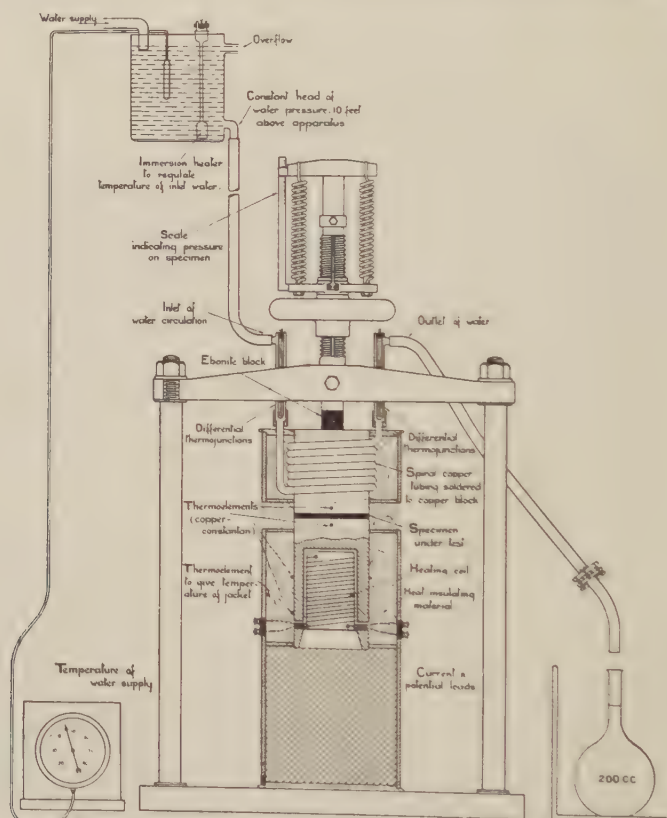
Fig. 4. Apparatus for heat transmission tests on thin discs.

* "Ueber einige Abbildungsaufgaben," *Crelle*, **70**, 105–120 (1869); and "Sul problema della temperature stazionarie," *Annali di Matematica*, **1**, 89 (1867).

† The reader who is interested in the theoretical aspect might be referred to an important paper by H. F. Moulton (*Proc. London Math. Soc.* **3**, 104) and also to the treatment in J. J. Thomson's *Recent Researches in Electricity and Magnetism*, p. 208, and Jeans' *Electricity and Magnetism*, 4th edition, p. 354.

ago and half a century has elapsed since Kirchhoff* showed the possibilities of the method in the solution of physical problems. The case worked out by Kirchhoff was that of a plate condenser in an infinite medium.

§ 3. APPARATUS FOR THE DETERMINATION OF THE CONDUCTIVITY OF THIN SHEETS

Data as to the heat transmission through thin sheets of electrically insulating materials are frequently required. Various forms of apparatus have been devised for these tests by Dr Kaye and the writer. In the one shown in Fig. 4 a copper rod $1\frac{3}{4}$ in. in diameter is hollowed out and a heating coil is inserted, as indicated in the figure. The specimen under test rests on the top surface of this rod and superimposed is a water-cooled rod pressed down by a spring. The top rod is surrounded on its outer surface by a copper spiral through which flows a steady stream of water. The temperature rise of the water is measured by a set of differential thermocouples. The surfaces of the copper bars are ground plane to an accuracy of $1/10{,}000$ in.

The essential features of a modification of this apparatus for use with thicker materials are shown in Fig. 5. In this arrangement the heating coil is in the form of a flat disc, and two samples of material are employed in the test.

Fig. 5.
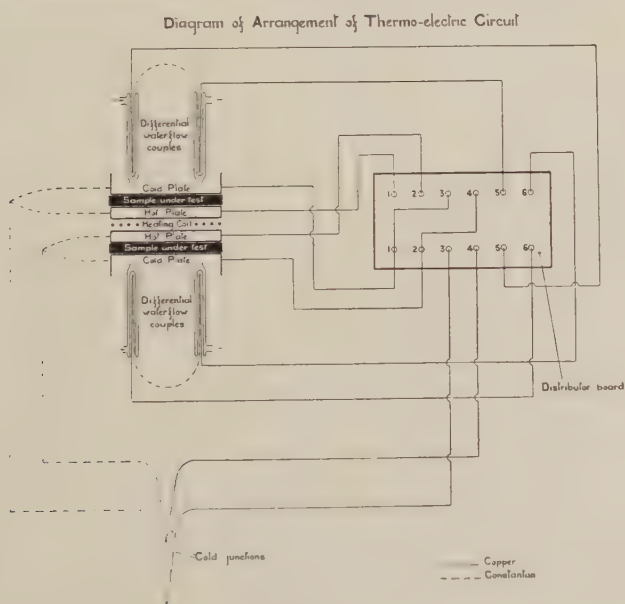
The "gradient along a bar" method was also used and found satisfactory for such materials as mica. This apparatus is shown in Fig. 6. One of the bars is provided with a heating coil and the other with a cooling coil, these being attached

* *Gesamm. Abhandl.* p. 112; *Berlin Akad. Monatsberichte*, p. 144 (1877).

to the extreme ends of the bars. Thermocouples were arranged at regular intervals along both bars so that the temperature gradient could be observed. The bars were supported vertically, the lower one being fixed while the top one was movable. In this apparatus the quantity of heat flowing into the specimen and out of it was obtained by observations of the temperature gradient along the copper bars and a knowledge of the thermal conductivity of the copper.

The two surfaces were amalgamated so that good thermal contact with the specimen was ensured. In order to determine the thermal resistivity of the two mercury films, which were always present when a specimen was interposed between the bars, some tests were made using a sheet of iron 0·002 inch thick instead of the specimen. From observations of the discontinuity in the temperature gradient across the mercury joints and the thermal conductivity of the iron specimen, the resistance of the film was calculated.

It might be mentioned that with this apparatus tests on a sample of mica of two thicknesses, one twice the other, gave very satisfactory agreement between the values of the thermal conductivity. It might also be mentioned that this apparatus is not well adapted for the study of poor heat-conductors

Fig. 6. Apparatus for the "gradient along a bar" method.

unless thin specimens be used. For if the thermal resistivity of the material be high, the temperature drop across the specimen is large and the gradient along the two bars small. As it is necessary to measure the gradient along the bar, this limits the use of this simple form of apparatus.

To study the variation in the conductivity over the area of a large sheet the apparatus shown in Fig. 7 was designed. The heating coil is clamped between two flat copper plates partially embedded in a block of ebonite: the amalgamated face is horizontal and downwards. The complete heating unit is carried on a spindle working in guides, and once it is properly aligned with the cold plate these guides ensure that the hot face registers correctly with the cold face. This automatic alignment of the two surfaces is essential when point to point tests have to be made on large sheets. The cold surface is the end of a cylindrical block of copper maintained at a constant temperature by the circulation of water. The temperature of the inlet water to the "cold" block is so adjusted that the mean of the inlet and outlet temperatures is equal to the temperature of the surrounding atmosphere: this arrangement eliminates the correction for loss or gain of heat from the block.
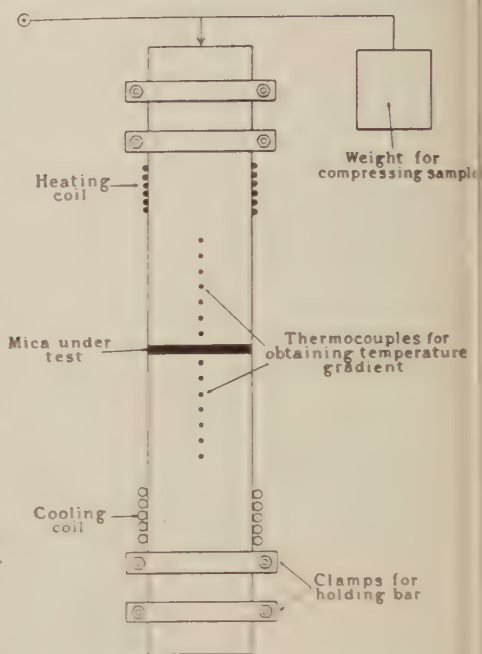
The spreading out of the lines of heat flow at the boundary is a point to which attention must be given. The two-dimensional case is amenable to mathematical solution, but not that of circular discs. Probably the simplest procedure if absolute values were desired would be to measure the capacity of a condenser with a sheet of dielectric between the plates.
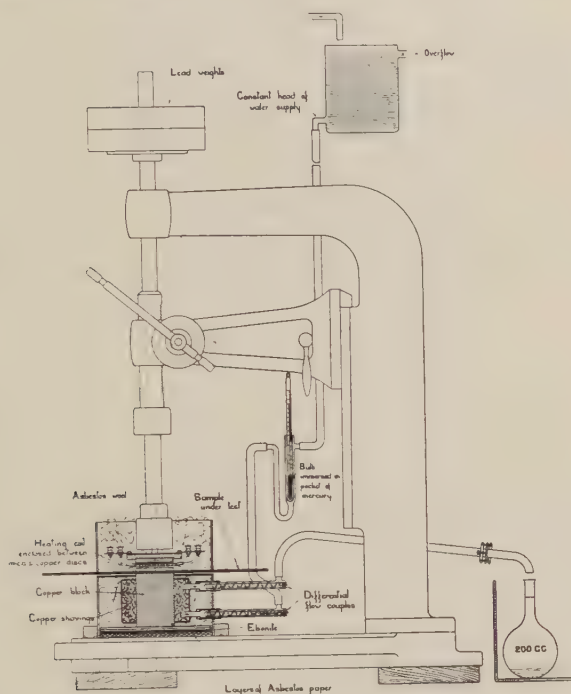

Fig. 7. Apparatus for measuring the thermal conductivity at various points on a sheet of material.

The question of the effect of pressure on the conductivity of material in laminated form is an important one, as, for example, in the case of laminated stampings used in electrical construction. The form of apparatus employed in this connection is shown in Fig. 8 and is self-explanatory. The apparatus is rigidly constructed to withstand the pressure applied by means of three springs. The hot and cold plates are 3 in. in diameter. Now this apparatus, although well adapted for the measurement of the heat transmission across the laminations of the block of stampings, is not suitable for the determination of the conduction along the stampings. For this test a bar about 15 in. long of square section with a 2 in. side was built up and an apparatus of the form shown in Fig. 11 used. One end of the bar was electrically heated, while the other end dipped into a pot of mercury carrying the cooling coil through which a measured flow of water circulated. The temperature distribution of the bar was found by means of thermoelements and the bar was well lagged. A detailed description of this type of apparatus is given later.
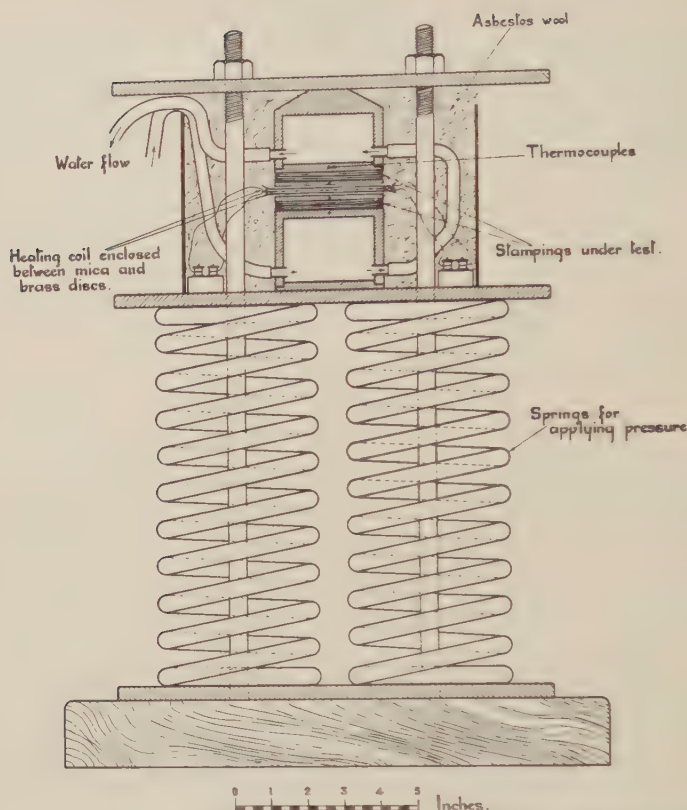
11

Fig. 8. Apparatus for tests on heat transmission through laminated materials under pressure.

§ 4. APPARATUS FOR THE DETERMINATION OF CONDUCTIVITY AT HIGH TEMPERATURES

The materials studied at high temperatures have been either refractories of high conductivity or good heat-insulating bricks employed as lining to the refractories. Mention may be made of two forms of apparatus.

In the apparatus shown in Fig. 9 a furnace is built up with "silit" heaters. These heaters are cylindrical rods of a refractory material and can be connected across the mains. A rod $\frac{3}{8}$ in. in diameter and 10 in. in length will dissipate about 500 watts in free air, the temperature of the surface being then about 1200° C.

The calorimeter and guard ring are made of thick slab brass with grooves milled in the back surface of each for water circulation. To reduce heat transfer between calorimeter and guard ring, the edges of the plates are chamfered off so that the edges on the brick are near together, whilst the rear parts are well separated. The brick rests on a thick flat plate of nichrome, a sheet of micanite being interposed to protect the thermojunction. The water flowing through the calorimeter is supplied from a constant-head tank.
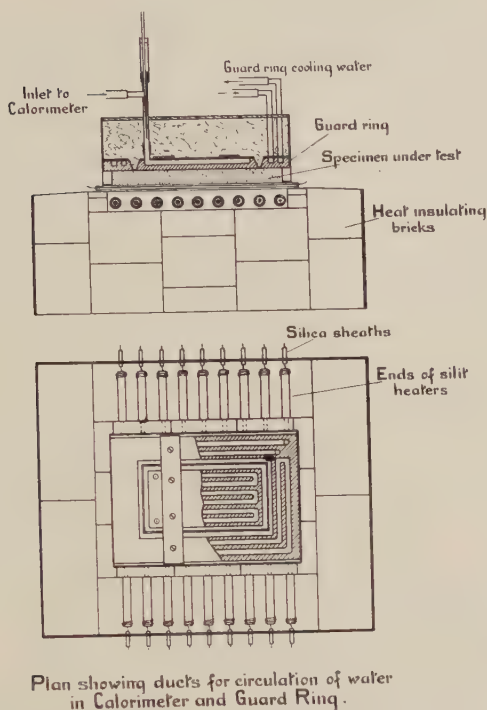
Fig. 9. Apparatus for thermal conductivity tests at high temperatures.

§ 5. APPARATUS FOR THE DETERMINATION OF THE THERMAL CONDUCTIVITY OF METALS AND ALLOYS

Most text-books in approaching the question of the thermal conductivity of metals deal at length with Forbes' method. I think the procedure introduces unnecessary difficulties into the subject and it would be better to approach the question from another angle. Forty years ago Berget described a guard tube method for the determination of the conductivity of metals and such a device greatly simplifies the theory of the method.

Take the case of the determination of the conductivity of a single crystal of aluminium with the apparatus shown in Fig. 10. The method of experiment is one in which a longitudinal heat flow is set up in a cylindrical bar 1·4 cm. in diameter and heat loss from the sides is prevented by the use of a guard tube, which in this case takes the form of a coaxial shield. The same gradient of temperature is maintained along the bar and the shield.

It is possible to dispense with one of the two heating coils and simply attach both specimen and guard tube to a cylindrical block containing a heating coil. This procedure simplifies the work, as the hot and cold ends of both the specimen and the guard tube are automatically maintained at temperature equality. In this particular apparatus the gradient in temperature along the coaxial tube was adjusted

to equality with that along the crystal by the aid of two thermocouples attached to the surface of the tube in positions corresponding to the two thermocouples on the aluminium crystal. The space between the aluminium crystal and the guard tube was filled with sil-o-cel, a very light powder of low thermal conductivity. The exterior surface of the guard tube was lagged with magnesia asbestos. The flow calorimeter was attached to the end of the crystal by Wood's metal.
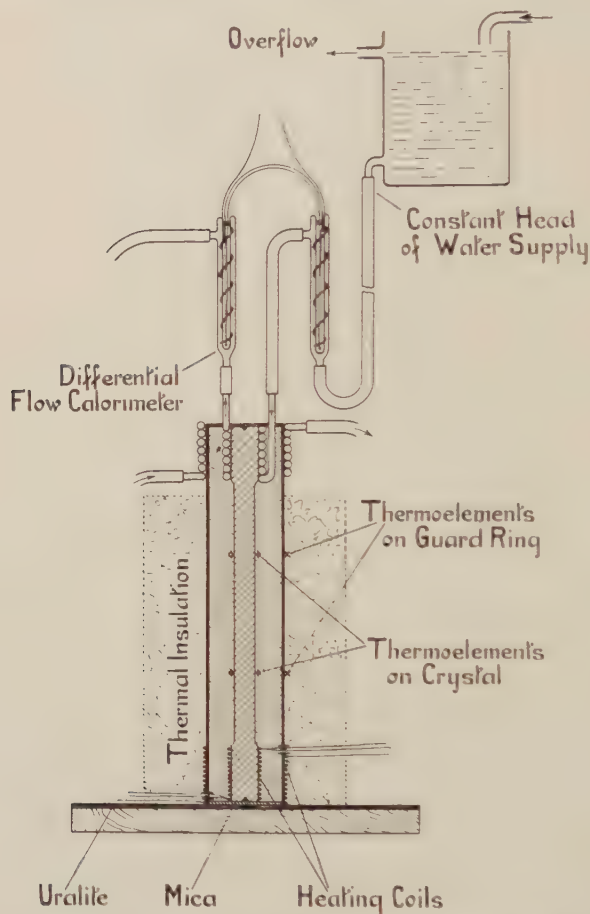
Fig. 10. "Guard tube" apparatus for tests on metal bars.

Specimens of as large a cross-sectional area as practicable should be selected. Although in the present instance the difficulties of making single crystals limited the diameter to 1·4 cm. in the experiments on bronzes and steels, bars up to 3·8 cm. diameter have been used.

The apparatus shown at the bottom of Fig. 11 was used by Mr Schofield and the writer to determine the conductivity of some aluminium and bronze alloys employed in the construction of zero engines. Both the thermal and electrical conductivities are measured at various temperatures in the range 80° to 300° C.

and the significant fact which emerged from the analysis of the results was that, with one or two exceptions, including notably the high silicon alloy, the Lorenz

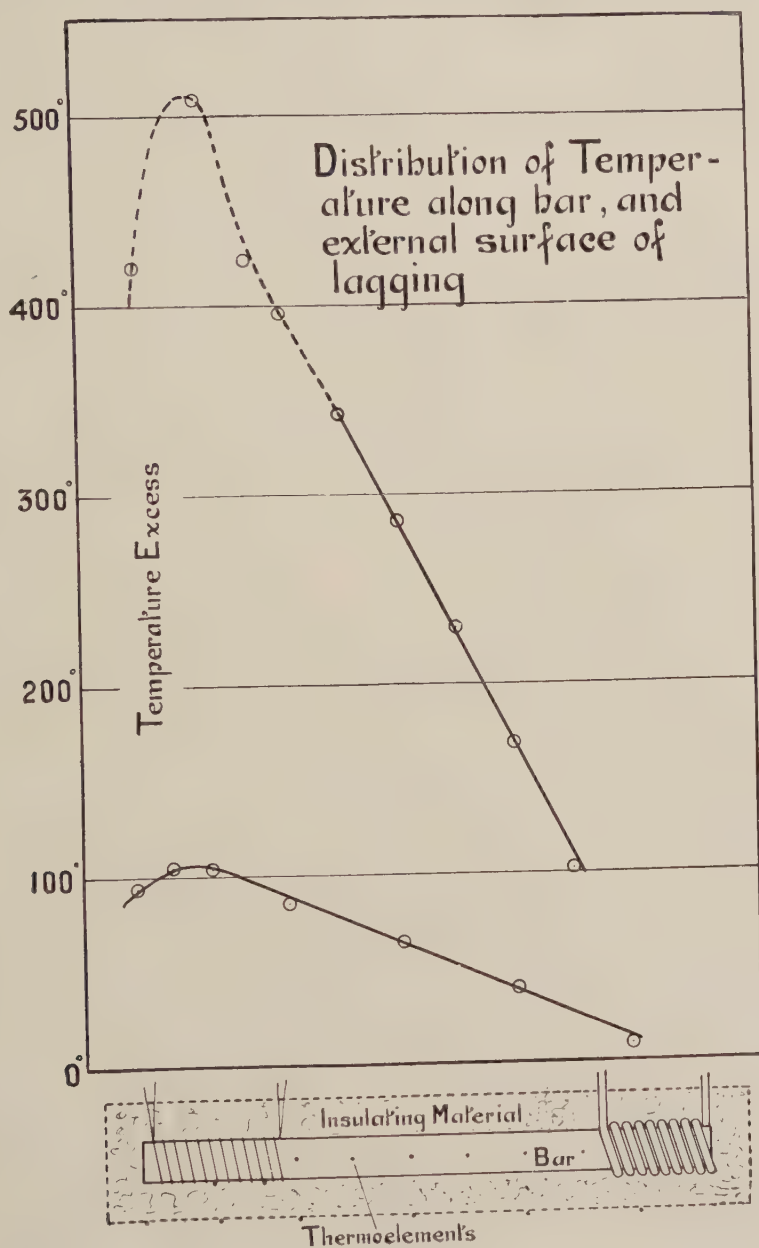Distribution of Temperature along bar, and external surface of lagging

Fig. 11.

law* applies approximately to the data obtained with these alloys. Furthermore it was found that the two groups of alloys had values of the Lorenz coefficient

* This law states that the function $k/\lambda T$ is approximately constant, where k is the thermal conductivity, λ the electrical conductivity and T the absolute temperature.

approximating to the values for the pure metals which formed the principal constituents, namely 5·5 for the aluminium alloys and 5·9 for the copper alloys. It is frequently stated in the literature of the subject that the Lorenz law fails when applied to data obtained on alloys. As a generalization this statement appears to be erroneous, and it is not improbable that the thermal conductivity data upon which it is based were unreliable.

It is well to point out that the values of the thermal conductivity of pure metals have not yet been finally settled. Purity of the material exerts a profound influence on the result, and the variations in Table 2 are to be ascribed to this cause, except in the case of Angell's results for high temperatures. He used a novel method: a stout bar was heated electrically and the temperature gradient from axis to surface was observed. In the case of aluminium it was too small to be measured accurately and furthermore longitudinal conduction disturbed the flow.

Table 2: Lorenz Coefficient for Aluminium

Authority	Purity of material % Al	Thermal conductivity at 100° C.	Lorenz coefficient × 10^{-9}				
			18°	100°	200°	300°	400°
Lees	99	—	5·1	—	—	—	—
Jaeger and Diesselhorst	99	0·49	5·2	5·4	—	—	—
Angell...	99	0·49	—	5·4	6·4	7·6	9·1
Konno	—	0·50	5·5	5·4	5·5	5·4	5·3
N.P.L.	99·7	0·53	—	5·3	5·6	5·8	6·0
,, ...	99·75	0·55	—	—	—	—	—
,, ...	Single crystal	0·55	5·4	—	—	—	—

Apparatus for the determination of the electrical conductivity of the alloys

It is essential to use the same specimens in the measurements of both thermal and electrical conductivities. In the study of the aluminium and bronze alloys the method used by Mr Schofield and the writer for the electrical measurement consisted in a comparison of the drop of potential across a measured length of the specimen with that across a standard resistance arranged in series with it, when a steady current of the order of 20 amp. was passed through the two. Into the specimen were pegged two thermocouples of copper-constantan, which served not only to measure the temperature but as potential leads for the resistance measurement. By appropriate switching arrangements the thermocouple wires could be rapidly altered so as to serve either purpose, as required. Since any temperature difference between the two ends of the bar would give rise to a thermal e.m.f. when measuring the p.d. across the bar, readings of the latter were obtained for the two directions of the current, so that, by taking the mean of these values, the effect of the thermal e.m.f. was eliminated. Effects, similar in character though smaller in magnitude, occurred in the potentiometer circuit itself and were corrected for by reversal of the potential leads and the leads supplying the current to the instrument.

Particulars of the apparatus are shown in Fig. 12. In order to economize time in testing the specimens, six were placed at once in the furnace, arranged in three pairs. As will be understood from Fig. 12, the three pairs of specimens, of which two pairs are shown, were contained inside a steel tube, and were separated from the tube and from each other by cross-pieces of "mabor" brick, through which their connecting leads passed. These leads were of copper 0·5 cm. in diameter, and were screwed into the ends of the specimen. The thermocouples, which were pegged into the bars, also passed through the mabor pieces. The assembled unit, consisting of the iron tube and its contents, was pushed into the furnace made up
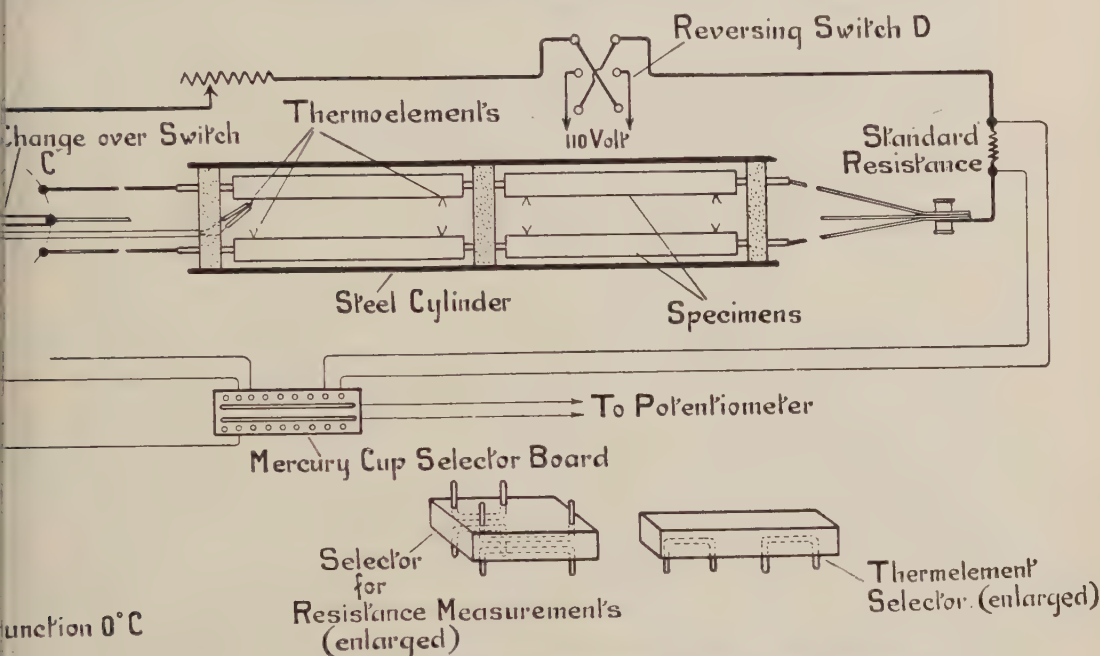

Fig. 12. Apparatus for measuring the electrical conductivity of alloys at high temperatures.

of a refractory tube 6 ft. long, which was wound with nichrome wire and well lagged. Auxiliary heaters of fireclay, wound with nichrome, were inserted into each end of the furnace. By appropriate adjustment of the three heating elements, a reasonable uniformity of temperature could be obtained inside the iron tube containing the specimens. The details of the electrical connections to the specimens are shown in Fig. 12. The change-over switch *C* allowed the steady current of 20 amp., supplied by a battery, to be passed through any one of the three pairs of specimens. This current could be reversed by the switch *D*. The potential leads from the standard resistance, and the thermocouple leads from the specimen, were connected to a mercury cup selector board, and thence to a potentiometer. When the thermocouple wires were used as potential leads for the resistance measurement, the selector block, which is shown (enlarged) near the bottom of Fig. 12, was employed. This consisted of copper rods mounted in ebonite. By insertion of the

pins on one side or the other of this block into the mercury cups, the connections of the potential leads from the specimens could be reversed; while, by the turning of the selector through 180° in the horizontal plane, either the pair of copper wires or the pair of constantan wires, forming the two thermocouples attached to the specimen, could be used as potential leads. When the thermocouple wires were being used for measurement of temperature, instead of resistance, the other selector shown in Fig. 12 was employed.

In determining the resistivity the usual procedure was to make measurements at a series of rising temperatures and to repeat the process until a steady state had been reached. The effect of this treatment on some of the alloys was to produce changes in structure, and the stable state was only attained after several heatings. Other alloys which had been thoroughly annealed gave very consistent results from the beginning. The measurements of thermal conductivity were made after the specimens had attained the steady state, as shown by the electrical measurements.

Discussion of results obtained in the study of the bronzes,
and a comparison with previous work

The main constituent in this series of alloys was copper, and a detailed analysis is set out in Table 3.

Table 3: Bronze Alloys

No. of bar	Percentage composition					
	Tin	Zinc	Phosphorus	Manganese	Aluminium	Copper. Remainder
1	5·0	2·0	0·15	—	—	92·8
2	10·0	2·0	0·15	—	—	87·8
3	8·0	—	0·30	—	—	91·7
4	0·5	38·5	—	0·30	—	60·7
5	10·0	2·0	—	—	—	88·0
6	—	—	—	—	10·0	90·0
7	12·4	—	0·4	—	—	87·2

Except in the case of alloy No. 4, which contains 38·5 per cent. of zinc, the total percentage of constituents other than copper has varied only from 7 to 13. Yet this comparatively small amount of dilution has produced a very marked effect on the conductivity, for whereas the value for pure copper at atmospheric temperature is 0·92 c.g.s., the corresponding values for the alloys range from about one-fifth to one-tenth of this amount.

Dealing first with the effect of tin, which is the most common constituent of the alloys, we may note that a comparison with the present series of experiments is afforded by the work of Lees*, of Grossmann†, of Schenck‡, and by a previous

* *Phil. Trans. Roy. Soc.* 208 A, 381–443 (1907).
† *Beibl. Ann. Physik*, 29, 4, 178–181 (1905).
‡ *Ann. Physik*, 32, 261 (1910).

test carried out by us (see Table 4). Approximate corrections have been applied to the values, where necessary, to reduce them to a common temperature of 20° C., and the results have been arranged in the table in order of the tin content of the alloy, an indication being also given of the amount of phosphorus present.

Table 4

| Date | Authority | Percentage of | | k_{20} |
		Tin	Phosphorus	
1908	Lees	0	0	0·918
1910	Schenck	5·0	0	0·172
1924	N.P.L. (No. 3)	8·0	0·3	0·108
1905	Grossmann	10·0	0	0·106
1920	N.P.L.	10·0	A small amount	0·100
1924	N.P.L. (No. 7)	12·5	0·4	0·087
1905	Grossmann	25·0	0	0·060
,,	,,	75·0	0	0·145
,,	,,	90·0	0	0·132
,,	,,	100·0	0	0·143

The above values are shown graphically in Fig. 13, which gives the relation between the tin content and the thermal conductivity. A thick line is drawn through the experimental points when they are not widely separated, according to percentage composition; and a dotted line, arbitrarily drawn, is used for the larger gap between the points representing 25 and 75 per cent. of tin. It will be seen that there is a satisfactory agreement between the various determinations.

In addition to the bars Nos. 3 and 7, results for which are included in the above table, bars Nos. 1, 2, 5 contain tin, but with an addition of 2 per cent. of zinc in each case. It will be observed from Table 5 that the conductivities of these bars do not differ very greatly from values to be expected from the tin content as read off from the curve in Fig. 13.

Table 5

| Specimen bar No. | Percentage of | | | k_{20} | |
	Tin	Zinc	Phosphorus	Observed	Estimated from tin content
1	5	2	0·15	0·188	0·17
2	10	2	0·15	0·101	0·10
5	10	2	—	0·115	0·10

A comparison of the observed values for bars Nos. 2 and 5, which are identical in composition except for a small amount (0·15 per cent.) of phosphorus, appears to show that the presence of the latter in minute quantities produces a very appreciable lowering of the conductivity. The figures for the two bars over the complete range from 75° to 250° C. differ on the average by 0·013 c.g.s., or by about

10 per cent. (see Fig. 14). In confirmation of this view of the effect due to phos-
phorus, it may be mentioned that according to Pfleiderer*, the conductivity of
copper at 30° C. (0·93 c.g.s.) was lowered to 0·25 and 0·12 respectively by the
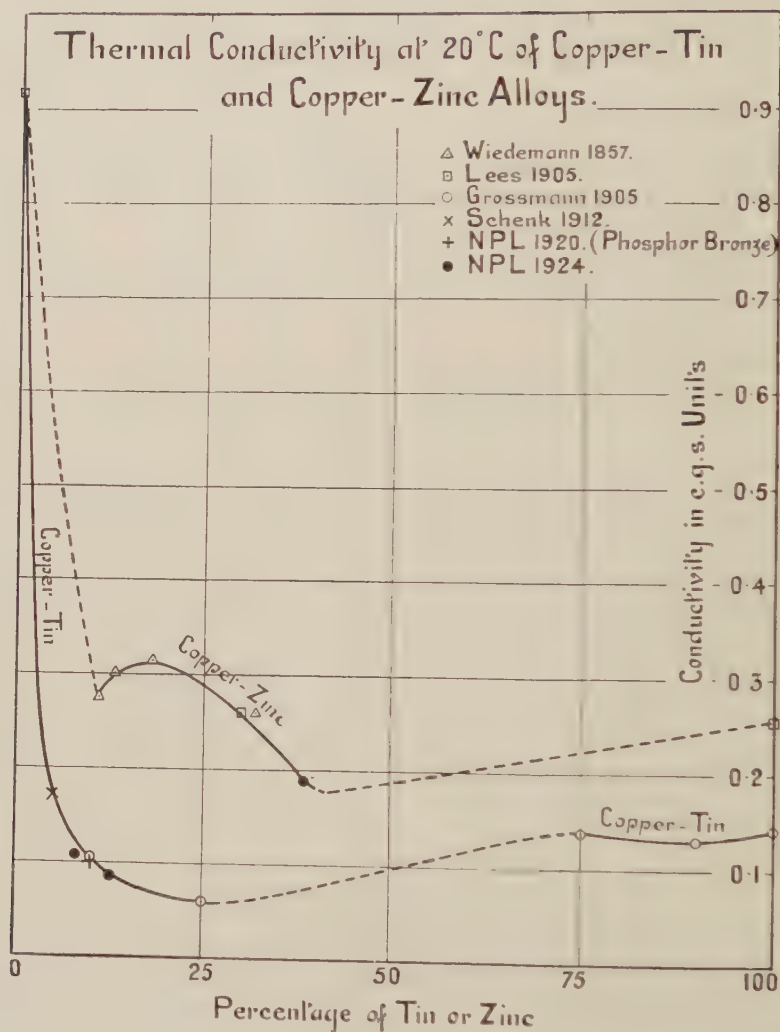presence of 0·63 and 1·98 per cent. of phosphorus.

Fig. 13.

Two other alloys remain to be considered (Nos. 4 and 6). For a comparison
with No. 4 the values obtained by Lees for brass (30 per cent. zinc), and by
Wiedemann† for alloys containing 11, 13, 18 and 32 per cent. of zinc, are given
in Table 6, and are plotted in Fig. 13. The figures of Wiedemann date from

* *Ges. Abhandl. Kenntniss der Kohle*, 4, 409 (1919).
† Landolt, *Phys. Chem. Tab.* (5th edition), 2, 1295 (1923).

1857, and should be accepted with some reserve. A curve has been drawn through all the values, but, for the reason stated, no special significance is to be attached to the inflection which it shows. The dotted portion of the curve is arbitrarily drawn and merely serves to connect points belonging to the same series.

Table 6

Authority	Percentage of zinc	k_{20}
N.P.L. (No. 4)	38·5	0·188
Lees, 1905	30·0	0·260
Wiedemann, 1857	11·0	0·275
,,	13·0	0·301
,,	18·0	0·313
,,	32·0	0·260
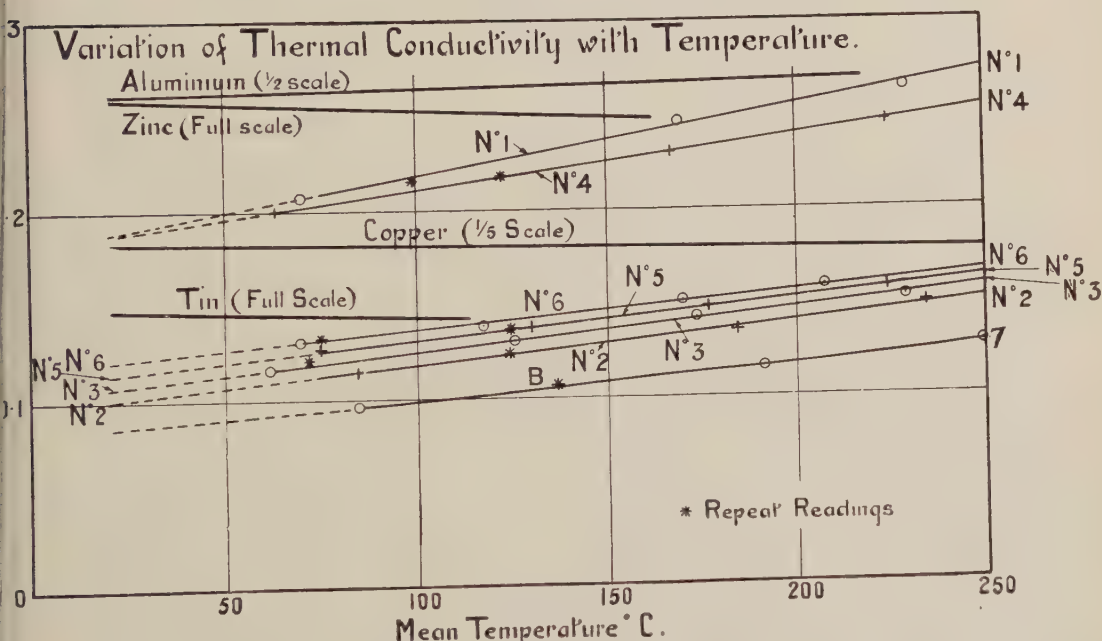
Fig. 14. The numbers denote the bronze alloys whose composition is given in Table 3.

For alloy No. 6 containing 10 per cent. of aluminium, no parallel figures have been traced, but it is perhaps worthy of note that the lowering of the conductivity of copper due to an admixture of 10 per cent. of aluminium is comparable with that due to the same amount of tin.

The above discussion has been concerned with the thermal conductivities of the alloys at one temperature (20° C.). The variations of conductivity with temperature are shown graphically in Fig. 14. For purposes of comparison the values for the pure metals composing the alloys are also indicated. It will be noticed that, whereas the variation of the pure metals is small, being on the whole negative

with rising temperature, the alloys show somewhat large positive coefficients. The aluminium crystal studied (99·6 per cent. Al, 0·2 per cent. Si, and 0·2 per cent. Fe) has a slight positive temperature coefficient—see Fig. 15. In all cases the relation between conductivity and temperature is approximately linear.
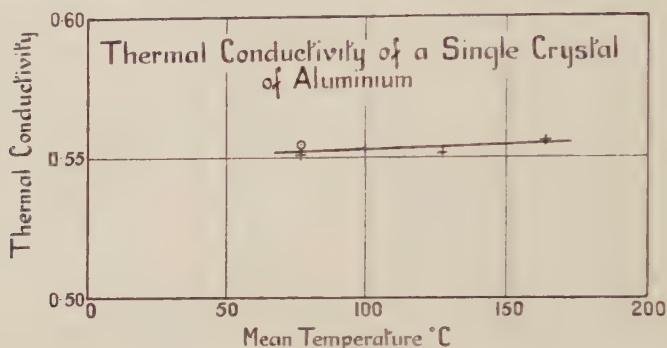

Fig. 15.

Relation between thermal and electrical conductivities for the bronze and aluminium alloys

The relation of the conducting powers of pure metals for heat and electricity has for a long time engaged the attention of physicists. In 1853 Wiedemann and Franz propounded a law to the effect that the ratio of the thermal and electrical conductivity was a constant for all metals. In 1872 Lorenz sought to prove, both on theoretical and experimental grounds, that this ratio was proportional to the absolute temperature. On the development of the electron theory of conduction, Drude, H. A. Lorentz, and J. J. Thomson have independently arrived at the same conclusion as Lorenz. The experimental evidence available up to 1900 was not sufficiently reliable to allow a definite confirmation of the theory. In that year Jaeger and Diesselhorst published the result of their investigation, which gave directly the ratio of the two conductivities for a number of pure metals over the range from 18° to 100° C. They showed that the value of the function $k \lambda T$ (Lorenz's constant) was nearly constant over this range, and did not vary greatly from metal to metal. In 1905 Lees obtained a close agreement with the values of Jaeger and Diesselhorst at 18° C., and carried the investigation down to − 170° C. Down to − 100° C. the value of the function showed, on the whole, a slight tendency to decrease, while below − 100° C. the decrease became more marked, and wide divergencies appeared among individual metals. Summarizing the position as regards the pure metals, it may be said that from − 100° C. to + 100° C., and probably to much higher temperatures, the function* $k \lambda T$ does not vary greatly in value from metal to metal and is nearly constant, showing a slight tendency to increase with temperature.

* In view of the fact that the function $k \lambda T$ is not an absolute constant, it would be preferable to use the term coefficient for the numerical value appropriate to particular metals and groups of alloys.

The position as regards alloys is not so clearly defined, since the very wide field of investigation which they present has hitherto hardly been explored at all.

The mean value of Lorenz's coefficient for all the copper alloys now under consideration, together with the percentage difference between the extreme values, is set out in Table 7.

Table 7

Temperature ° C.	Lorenz's constant, $k/\lambda T$	
	Mean value for all the bronze specimens	Percentage difference between extreme values
20	$[6 \cdot 1_5] \times 10^{-9}$	6
75	$5 \cdot 9_2$	5
100	$5 \cdot 8_7$	4
150	$5 \cdot 7_9$	2
200	$5 \cdot 7_8$	2
250	$5 \cdot 8_1$	2

Note. The value at 20° C. is obtained by extrapolation.

As regards the range covered by experiment (75° to 250° C.), it appears that the mean value of the function $k/\lambda T$ is practically constant, and that the extreme range of values for all the bronze bars does not exceed $\pm 2 \cdot 5$ per cent., an amount which is probably within the limits of experimental error. As to the absolute value of the function, it is to be noted that Jaeger and Diesselhorst's mean value for eight pure metals at 20° C. and 100° C. is approximately $5 \cdot 7_5$. Lees gives a similar value for the pure metals at 20° C., while his value for brass (which corresponds to alloy No. 4) is $5 \cdot 83$. I have studied a single crystal of very pure copper and obtained the value $5 \cdot 89$.

A comparison is also afforded by the work on the copper-tin alloys of Grossmann, whose values are set out in Table 8.

Table 8

Tin content (%)	Value of $k/\lambda T$
10	$5 \cdot 9_5 \times 10^{-9}$
25	$6 \cdot 3_8$
75	$5 \cdot 6_3$
90	$5 \cdot 4_0$
100	$5 \cdot 8_8$

The conclusion, which has been reached above, that all the bronzes under investigation obey Lorenz's law of the relation between thermal and electrical conductivity, has a practical bearing in that it means that the thermal conductivity could be deduced from a measurement of the electrical conductivity. The determination of this latter constant presents far less difficulty than that of the former, so that, if the assumption be made that Lorenz's law applies to other alloys of an analogous nature, their thermal conductivities could be deduced with comparative

ease by the method indicated. Since the temperature coefficients of thermal conductivity for the alloys do not differ very markedly, a rough value at any temperature could be obtained from a single measurement of electrical conductivity at atmospheric temperature, but a better approximation would be obtained if the electrical conductivity were determined at the appropriate temperature.

As regards the Lorenz coefficient for the aluminium alloys, with the exception of the 13 per cent. silicon alloy, the 21 alloys give very consistent results*, the mean values being as follows:

Table 9

Temperature (° C.)	Lorenz coefficient
80	$5 \cdot 3_1 \times 10^{-9}$
150	$5 \cdot 3_8$
200	$5 \cdot 4_9$

A determination of the Lorenz coefficient for a single crystal of aluminium (99·6 per cent. aluminium) gave the value $5 \cdot 46 \times 10^{-9}$, which is in good agreement with the mean value for this group of alloys, with the exception of the 13 per cent. silicon alloy. It will be observed that the Lorenz coefficient for the aluminium alloys is decidedly lower than the value for the copper alloys.

§6. ACKNOWLEDGMENTS

DISCUSSION

Dr G. W. C. Kaye: I wish to support Dr Griffiths' plea that methods which are primarily of historical importance should not be given the premier place when the principles underlying the measurement of thermal conductivity are taught. [The speaker then gave some results which he and Mr Higgins had obtained by the divided bar method on quartz crystal cut parallel and perpendicular to the axis.]

* Since this work was completed a paper by Masumoto, *Sci. Rep. Tôhoku Imp. Univ.* 13, 229 (1925), has come to my notice which deals with some eighteen aluminium alloys. His data indicate that the mean value of the Lorenz coefficient is $5 \cdot 4 \times 10^{-9}$ at 30° C., which is in satisfactory accord with the above data.

As regards the electrically heated plate method, it has the additional advantage of being convenient for exploring the variation of thermal conductivity with temperature. The true conductivity is obtained by jacketing the entire apparatus in a constant temperature enclosure and working with only small differences of temperature between the hot and cold plates. [The speaker illustrated this point by showing results for single bismuth crystals, water, various oils and solid and liquid sulphur.]

Mr F. H. Schofield dealt with the application of the method of conformal representation to a problem mentioned by Dr Griffiths—namely, the heat flow from a thin plate embedded in a mass of poorly conducting material, which is contained within a rectangular isothermal boundary. He gave an expression for the "edge correction" for such a plate and showed a diagram of the calculated distribution of flow lines and isothermals within the material.

Mr J. H. Awbery: I think the date is far distant when the theory of conformal representation will be capable of generalisation for solving problems in three dimensions. This is not to say that such problems are at present insoluble, but the method referred to—which is merely one form of the method of conjugate functions —is hardly likely to be applicable to them. We are accustomed to the idea of conformal transformations, in which one surface, say that of a sphere, is applied to another. In using the method for potential problems, we apply a polygon or other figure to a square, so that there is a one-one correspondence between points in the two figures. For a spatial problem, it would be necessary to transform a solid figure to a cube. The equations would be

$$x' = f_1(x, y, z); \quad y' = f_2(x, y, z); \quad z' = f_3(x, y, z).$$

The advantage possessed by the plane is that these equations become one, instead of two, with one variable only. In fact the new complex variable $(x + iy)$ is related to the old by one equation $(x' + iy') = f(x + iy)$. The method is only made possible by the dissymmetry between i and 1, a dissymmetry not shared by the three unit vectors in space.

Turning to the questions suggested by the lecturer in regard to the Lorenz function, I would ask whether we have real grounds for expecting a constant at all? Mica, sulphur and other non-metals have a thermal conductivity which is not infinitely small compared with metals, and this suggests that the atoms, as opposed to the electrons, must play some part in the thermal conductivity, although not in the electrical. After all, theory discusses only the relation between the electrical conductivity and the thermal conductivity due to the electrons, considered in the old theory as a perfect gas, and in the new one as a degenerate gas, whilst experiment gives the total conductivity. The idea that the metallic part (λ_m) and non-metallic part (λ_i) of the thermal conductivity could be separated occurred to Gruneisen and Goens*, but, if I have read them correctly, they make a curious

* *Zs. für Phys.* **44**, 615 (1927).

error in the treatment of the very valuable data which they give. They consider that the thermal *resistance* is additive, so that the total resistance is made up of that due to the electrons, and that due to the atoms. With the substitution of con- ductivity for resistance, I think their method will be very valuable. They give data for a number of specimens of differing purity, and if the thermal conductivity be plotted against the electrical conductivity, the various specimens give points along a curve. This curve can be extrapolated to the point where the electrical conductivity is zero, and it will be found that this point is not at the origin. When the electrical conductivity ceases, so that electrons are presumably taking no part in the pheno- mena, there is still a finite thermal conductivity. I would suggest that this residuum is λ_i and that if it were subtracted from each observed λ so as to give λ_m, we should find that $\lambda/\sigma T$ was much more nearly constant. On this view, the constancy of the Lorenz coefficient for a series, say the aluminium series, of alloys is compre- hensible. Whilst the alloy remains rich in aluminium, λ_i is small compared with λ_m, so that the coefficient is nearly the theoretical one.

All the above argument really supposes a homogeneous medium. In the case of a metal made up of separate crystals, it may be that there are temperature dis- continuities at the faces, although this is not certain. It is also possible that in alloys there is a counter e.m.f. proportional to the current, and therefore indis- tinguishable from a resistance; there would be no counterpart to this in the thermal conductivity, so that Lorenz' coefficient would again be altered. This idea was worked out by Rayleigh*.

Mr R. W. POWELL: I was pleased to hear Dr Griffiths refer to some of the varia- tions which are found in the values of the Lorenz constant for different metals, and in view of the theoretical interest attached to this function I would like to emphasise the importance of combining a measurement of electrical conductivity with that of thermal conductivity. Dr Griffiths has shown us an example in which the thermal conductivity of the metal under test varied according to its previous heat treatment, from which we see that the two conductivities should be measured simultaneously in order to give the correct value for the Lorenz constant.

I would also like to mention that Worthing† has determined the Lorenz constant for carbon, in the form of an electrically heated filament, and has found that the value decreases from $2 \cdot 8 \times 10^{-8}$ ohm-calorie units at 1700° absolute to $2 \cdot 1 \times 10^{-8}$ ohm-calorie units at 2100° absolute. The Lorenz constant for carbon thus appears to have a value about four times that for the metals, and decreases with increasing temperature, whereas the opposite change appears to be indicated with metallic conductors.

Mr S. SKINNER: I feel that we are much indebted to Dr Ezer Griffiths for a very complete account of the methods for measuring thermal conductivities at the National Physical Laboratory, and I think we now have accurate values of the thermal conductivity and of the temperature coefficients for many materials used

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* *Nature*, 54, 154 (1896).   † *Phys. Rev.* 4, 535 (1914).

in the arts for heat insulators and other purposes. This was badly required, as anyone may see who examines the older text-books. It is a feature of the methods that great pains have been taken to make the conditions thoroughly definite, which was not the case with many of the older methods. The analogy between heat conduction and the flow of electrical currents is of great use. I look forward to further good results from the continuance of these measurements.

Dr ALLAN FERGUSON: I am sure that we have listened to Dr Griffiths' critical address with both pleasure and profit—he has given an admirably clear account of a successful attack on a subject which bristles with experimental difficulties. I should like to take up the cudgels in defence of Forbes' memory—his method, perhaps unduly exalted in the past, is liable to a period of undue depreciation. But it stands as a highly ingenious attempt to obtain absolute values for K at different particular temperatures which shall be quite independent of any assumptions concerning the variation of conductivity with temperature. If we consider any cross-section at a temperature θ on a long bar, the conductivity K at that temperature is unambiguously determined if we know the sectional area and temperature at the point in question and the rate of flow of heat across the isothermal surface at this point, assumed plane. This last quantity is of course obtained from the "dynamical" experiment, which may not be above criticism. *A propos* of such experiments, it would be very helpful to have Dr Griffiths' opinion of the "five-fourths" power law of radiation, inasmuch as this law enables one to obtain a solution of certain of the conductivity equations which is valid over a range considerably wider than is usually assumed possible. In my own experience, a plot of $\theta^{-\frac{1}{4}}$ against the time yields a curve which is rectilinear over a surprisingly wide range. A number of the methods described eliminate side losses as much as possible, and we should welcome remarks from Dr Griffiths' dealing with the relative merits of methods which seek to *eliminate* these losses and of those methods which seek to *determine* these losses on the basis of the $\frac{5}{4}$ths or some other suitable law of radiation.

Where Dr Griffiths has given so much it seems selfish to ask for more, but there is a large class of experimental methods which Dr Griffiths has but touched upon. I refer to those methods in which the conductivity equation is integrated over a suitable range of temperature—usually with simplifying assumptions concerning the variation of conductivity with temperature. Examples of such methods are afforded by two concentric spherical surfaces or two coaxial cylinders maintained at temperatures θ_1 and θ_2 respectively.

Lord Kelvin was wont to speak enthusiastically of Ångström's method, and one or two experimenters, employing ingenious variations in the method of controlling the heat supply, have lately used it. A few remarks from Dr Griffiths on these methods would add greatly to the value of his already valuable address.

Mr T. SMITH: I do not think the suggestion that these problems in three dimensions are insoluble should pass without comment. Much work has been done in recent

years on orthogonal space transformations, and some of the results may be applicable to physical problems. Even if we take the most unfavourable view, and assume that a solution in tabulated analytical functions is unattainable, we can always reach a solution to any desired degree of accuracy by numerical processes. Given a definite problem of this type, such as is of importance in experimental work, a solution, at least as accurate as is significant when experimental errors are considered, can always be obtained in a reasonably short time by the continuous refinement of a crude first approximation. Perhaps the most remarkable fact about such solutions of some problems is the rapidity with which a sufficiently correct solution may be reached.

The PRESIDENT: The treatment and form of the subject of thermal conductivity has changed greatly of recent years. Formerly it was very academic, a source of Fourier and conformal representation problems, and of laboratory experiments like the Forbes' bar experiment. To-day, as can be seen by a glance at the exhibits on the table, the subject has many practical applications. Almost the only industrial problems of the academic days was the flow of heat through boiler plates, and to this the physicist's results were applied timidly. Nowadays refrigeration and other industrial processes call for precise knowledge of materials. Industry has thus compelled the development of new and rapid methods of measurement which, as the lecturer has shown us, can be accurate, simple and beautiful. Provided that the pressure of industrial demands does not overwhelm the study of the abstract side of thermal conductivity, the new conditions are greatly to the advantage of pure physics.

LECTURER'S REPLY: Mr Schofield by the application of conformal representation has shown how the edge effect can be calculated exactly; hitherto workers have depended on the method of Fourier analysis for a close approximation. It must be pointed out, however, that his solution applies to the case of an infinitely long strip but not to a circular plate or to the heat leakage at the four corners of a rectangular plate.

I agree with Mr Skinner that electrical methods, such as the determination of the capacity of a condenser of appropriate form, can be employed for ascertaining the coefficient in calculations of heat flow for geometrical forms too complicated to tackle mathematically.

It is gratifying to hear from Mr T. Smith that some of the problems mentioned in the lecture might be amenable to solution by approximate methods giving any desired degree of accuracy.

In spite of Dr Ferguson's defence of Forbes' method, I am afraid I am still of opinion that any merit it may possess lies in its potentialities as an examination question. As regards the five-fourths power law for heat loss by convection, it appears to hold good for natural convection when there are no draughts. This is a subject I am now studying in connection with heat transmission to or from pipes in an air stream. In the methods indicated by Dr Ferguson the differences of

temperature are comparatively small and it suffices generally to assume a linear variation of conductivity with temperature, which will usually yield a simple formula. I have not referred specifically to methods employing coaxial cylinders or concentric spheres, since it seems to me that the theoretical advantages obtained by the use of such shapes in place of plane slabs and the like are outweighed by the practical difficulties involved.

I greatly appreciate the kind remarks of the President and other Fellows. All I have attempted in the lecture is to indicate the practical difficulties of the worker who has to determine the thermal conductivity of a material in whatever form it may be supplied to him.
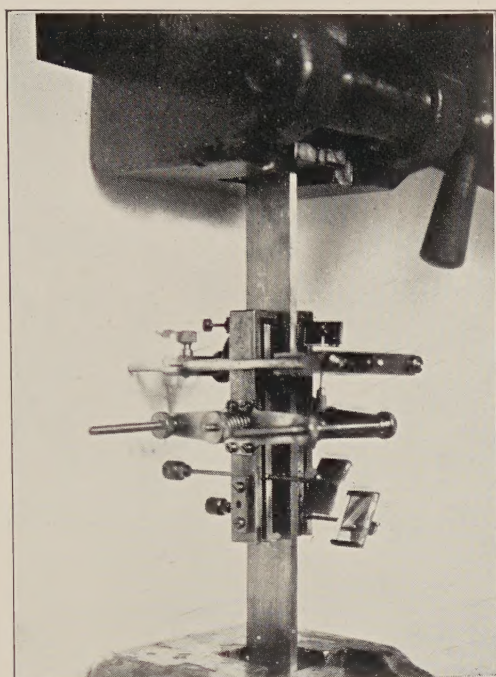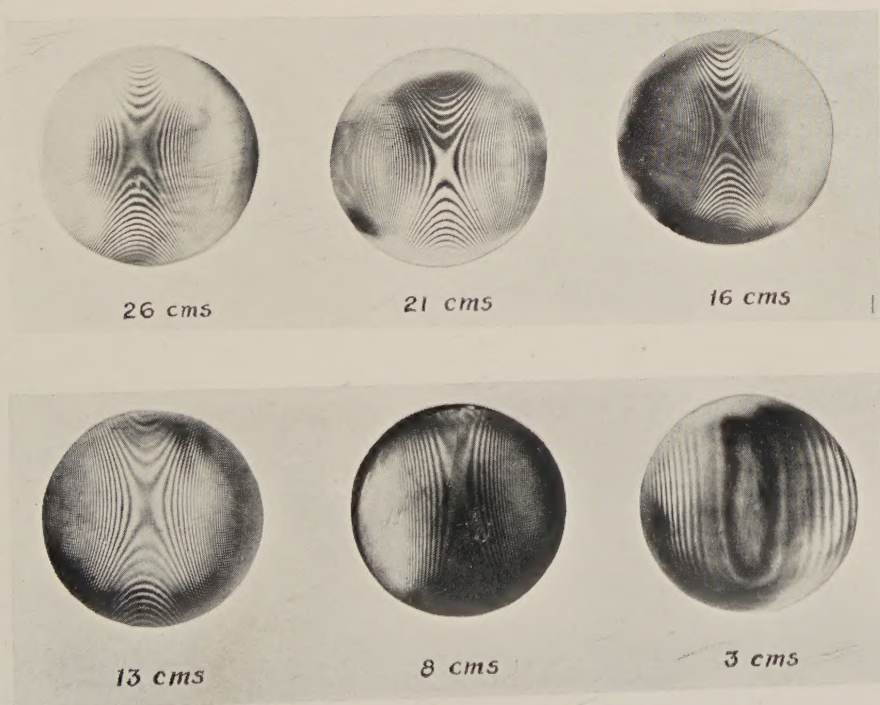
Fig. 6. Extensometers used in testing-machine

26 cms   21 cms   16 cms

13 cms   8 cms   3 cms

Interference fringes on bent glass plate

(Length of plate, 60 cm. Knife-edges 20 cm. apart. Distances from end are given.)